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MATH 101
HOMEWORK 1 SOLUTIONS

1. Find $\lim\limits_{x\to 1} \dfrac{x^3+x^2+x-3}{x^3+2x^2-x-2}$.

Solution:

$$\begin{aligned}
\lim_{x\to 1} \frac{x^3+x^2+x-3}{x^3+2x^2-x-2} &= \lim_{x\to 1} \frac{(x-1)(x^2+2x+3)}{(x-1)(x^2+3x+2)} \\
&= \lim_{x\to 1} \frac{(x^2+2x+3)}{(x^2+3x+2)} \\
&= 1.
\end{aligned}$$

2. Find $\lim\limits_{x\to 1} \dfrac{x^3-3x^2+5x-3}{x^3-4x^2+5x-2}$.

Solution:

$$\begin{aligned}
\lim_{x\to 1} \frac{x^3-3x^2+5x-3}{x^3-4x^2+5x-2} &= \lim_{x\to 1} \frac{(x-1)(x^2-2x+3)}{(x-1)^2(x-2)} \\
&= \lim_{x\to 1} \frac{(x^2-2x+3)}{(x-1)(x-2)} \\
&= \text{does not exist.}
\end{aligned}$$

3. Solve the following problems from the book:

- Page 77, Exercises 6, 8, 10.

6) Equation of the line through $(3,3)$ and $(-2,5)$ is $y = \frac{5-3}{-2-3}(x-3)+3 = -\frac{2}{5}(x-3)+3$. Or equivalently $2x+5y=21$.

8) Equation of line through $(3,1)$ and parallel to $2x-y=-2$. Rewriting the line $2x-y=21$ as $y=2x+2$, we see that its slope is 2. The line which passes through $(3,1)$ must therefore have slope 2. Its equation is $y=2(x-3)+1$.

10) Equation of line through $(-2,-3)$ and perpendicular to $3x-5y=1$. The slope of the line $3x-5y=1$ is $3/5$, therefore the slope of a line perpendicular to it is $-5/3$. The required equation is $y=(-5/3)(x+2)-3$.

- Page 79, Exercises 94, 96, 98, 100.

94) $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}}$. Here we are using the table on page 51 about the domain and the range of the inverse trigonometric functions.

96) $\sec(\tan^{-1} 1 + \csc^{-1} 1) = \sec(\pi/4 + \pi/2) = \sec(3\pi/4) = -\sqrt{2}$.

98) If $\alpha = \sec^{-1}(y/5)$, then $\cos\alpha = 5/y$. Then $\tan\alpha = \pm\sqrt{y^2 - 25}/5$, where the sign is $+$ if $y > 0$, and $-$ if $y < 0$. This follows from the graph in figure 50-d on page 52.

100) If $\alpha = \tan^{-1} x/\sqrt{x^2+1}$, then $|\alpha|$ can be put into a right triangle with adjacent side $\sqrt{x^2+1}$ and opposite side $|x|$. The hypothenuse becomes $\sqrt{2x^2+1}$, and $\sin\alpha = x/\sqrt{2x^2+1}$.

- Page 81, Exercise 8.

The coordinates of P is given by $P = (a/2, b/2)$. The slope of the line through the origin and P is therefore b/a. The slope of the line AB is $-b/a$. We have $OP \perp AB$ when $(b/a)(-b/a) = -1$, or equivalently when $a = b$, since both a and b are positive.

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