18 March 2004

MATH 101
HOMEWORK 2 SOLUTIONS

p157 Ex-4. Find $f'(x)$ using the definition of derivative and then evaluate $f'(-3)$, where $f(x) = x + 9/x$.

Solution:
$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{[(x+h) + 9/(x+h)] - [x + 9/x]}{h} = \lim_{h\to 0} \dfrac{x^2 + xh - 9}{x(x+h)} = 1 - 9/x^2$, so $f'(-3) = 0$.

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p157 Ex-7 Find the first and second derivatives of $y = x^2 + x + 8$.

Solution:
$y' = 2x + 1$, $y'' = 2$.

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p157 Ex-8 Find the first and second derivatives of $s = 5t^3 - 3t^5$.

Solution:
$s' = 15t^2 - 15t^4$, $s'' = 30t - 60t^3$.

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p157 Ex-9 Find the first and second derivatives of $y = \dfrac{4x^3}{3} - 4$.

Solution:

$y' = 4x^2$, $y'' = 8x$.

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p157 Ex-10 Find the first and second derivatives of $y = \dfrac{x^3 + 7}{x}$.

Solution:

$y' = \dfrac{(3x^2)(x) - (x^3 + 7)(1)}{x^2} = \dfrac{2x^3 - 7}{x^2} = 2x - \dfrac{7}{x^2}$, $y'' = 2 + \dfrac{14}{x^3}$.

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p169 Ex-1 $s = t^2 - 3t + 2, \quad 0 \le t \le 2.$

Solution:

(a) Displacement$= \Delta s = s(2) - s(0) = -2m$. $v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{-2m}{2sec} = -1m/sec.$

(b) $v = s' = 2t - 3$, speed at the end points $|v(0)| = 3m/sec$, $|v(2)| = 1m/sec$. Acceleration $a = v' = 2m/sec^2$, acceleration at the end points $a(0) = 2m/sec^2$, $a(2) = 2m/sec^2$.

(c) The body changes direction when v changes direction. For this first v must be zero. $v = 2t - 3 = 0$ when $t = 3/2sec$. For $0 \le t \le 3/2$, v is negative and for $3/2 \le t \le 2$ v is positive. So the body changes direction when $t = 3/2sec$.

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p169 Ex-2 $s = 6t - t^2, \quad 0 \le t \le 6.$

Solution:

(a) Displacement$= \Delta s = s(6) - s(0) = 0$. $v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{0}{6sec} = 0m/sec.$

(b) $v = s' = 6 - 2t$, speed at the end points $|v(0)| = 6m/sec$, $|v(6)| = 6m/sec$. Acceleration $a = v' = -2m/sec^2$, acceleration at the end points $a(0) = -2m/sec^2$, $a(6) = -2m/sec^2$.

(c) The body changes direction when v changes direction. For this first v must be zero. $v = 6 - 2t = 0$ when $t = 3sec$. For $0 \le t \le 3$, v is positive and for $3 \le t \le 6$ v is negative. So the body changes direction when $t = 3sec$.

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p169 Ex-3 $s = -t^3 + 3t^2 - 3t, \quad 0 \le t \le 3.$

Solution:

(a) Displacement$= \Delta s = s(3) - s(0) = -9m$. $v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{-9m}{3sec} = -3m/sec.$

(b) $v = s' = -3t^2 + 6t - 3$, speed at the end points $|v(0)| = 3m/sec$, $|v(3)| = 12m/sec$. Acceleration $a = v' = -6t + 6$, acceleration at the end points $a(0) = 6m/sec^2$, $a(3) = -12m/sec^2$.

(c) The body changes direction when v changes direction. For this first v must be zero. $v = -3t^2 + 6t - 3 = -3(t-1)^2 = 0$ when $t = 1sec$. For all other values of t the velocity is negative, so the body never changes direction.

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p169 Ex-7 The equations for free fall at the surfaces of Mars is $s = 1.86t^2$, and on the surface of Jupiter is $s = 11.44t^2$. Here s is in meters and t is in seconds. How long does it take a rock falling from rest to reach a velocity of $27.8m/sec$ on each planet?

Solution:

On Mars $s = 1.86t^2$, $v = 3.72t$. Solving $3.72t = 27.8$ gives $t = 7.4sec$.

On Jupiter $s = 11.44t^2$, $v = 22.88t$. Solving $22.88t = 27.8$ gives $t = 1.2sec$.

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p169 Ex-9 On an airless planet a ball is shot upwards with an initial velocity of $15m/sec$. The ball reached its maximum height $20sec$ after launch. The ball's motion is governed by the formula $s = 15t - (1/2)g_s t^2$ where g_s is the acceleration of gravity. What is the value of g_s on this planet?

Solution:

$s = 15t - (1/2)g_s t^2$, $v = 15 - g_s t$. When the maximum height is reached the velocity is zero. Solving $v = 15 - g_s t = 0$ we get $t = 15/g_s$. Therefore $15/g_s = 20$ gives $g_s = (3/4)m/sec^2$.

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