Math 101 Homework-6 Solutions
May 2004

Page 564, Exercise 23:

$$\frac{y^2+2y+1}{(y^2+1)^2} = \frac{Ay+B}{y^2+1} + \frac{Cy+D}{(y^2+1)} = \frac{1}{y^2+1} + \frac{2y}{(y^2+1)^2}.$$

$$\int \frac{y^2+2y+1}{(y^2+1)^2}\,dy = \int \frac{1}{y^2+1}\,dy + \int \frac{2y}{(y^2+1)^2}\,dy = \arctan y - \frac{1}{y^2+1} + C.$$

Page 564, Exercise 26:

$$\frac{s^4+81}{s(s^2+9)^2} = \frac{A}{s} + \frac{Bs+C}{s^2+9} + \frac{Ds+E}{(s^2+9)^2} = \frac{1}{s} + \frac{-18s}{(s^2+9)^2}.$$

$$\int \frac{s^4+81}{s(s^2+9)^2}\,ds = \int \frac{1}{s}\,ds - 18\int \frac{s}{(s^2+9)^2}\,ds = \ln|s| + \frac{9}{(s^2+9)} + C.$$

Page 601, Exercise 50:

Let $u = \cos^{-1}\left(\frac{x}{2}\right)$, and apply by parts to obtain

$$\begin{aligned}
\int \cos^{-1}\left(\frac{x}{2}\right)\,dx &= x\cos^{-1}\left(\frac{x}{2}\right) + \int \frac{x\,dx}{\sqrt{4-x^2}}, \quad \text{Now let } y = 4-x^2,\\
&= x\cos^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\int \frac{dy}{\sqrt{y}}\\
&= x\cos^{-1}\left(\frac{x}{2}\right) - \sqrt{y} + C\\
&= x\cos^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + C
\end{aligned}$$

Page 601, Exercise 52:

Apply tabular integration to obtain

$$\begin{array}{l}
\qquad\quad \sin(1-x)\\
x^2 \xrightarrow{(+)} \cos(1-x)\\
2x \xrightarrow{(-)} -\sin(1-x)\\
2 \xrightarrow{(+)} -\cos(1-x)\\
0
\end{array}$$

This gives $\int x^2\cos(1-x)\,dx = x^2\cos(1-x) + 2x\sin(1-x) - 2\cos(1-x) + C.$

Page 601, Exercise 54:

Let $I = \displaystyle\int e^{-2x} \sin 3x \, dx$. Apply by parts with $u = \sin 3x$ to obtain

$$I = -\frac{1}{2} e^{-2x} \sin 3x + \frac{3}{2} \int e^{-2x} \cos 3x \, dx.$$

In the last integral apply by parts with $u = \cos 3x$ to obtain

$$I = -\frac{1}{2} e^{-2x} \sin 3x + \frac{3}{2} \left[-\frac{1}{2} e^{-2x} \cos 3x - \frac{3}{2} \int e^{-2x} \sin 3x \, dx \right] = -\frac{1}{2} e^{-2x} \sin 3x - \frac{3}{4} e^{-2x} \cos 3x - \frac{9}{4} I.$$

Solving for I gives $I = -\dfrac{2}{13} e^{-2x} \sin 3x - \dfrac{3}{13} e^{-2x} \cos 3x + C.$

`Comments to sertoz@bilkent.edu.tr`