Math 101 Homework-7 Solutions
May 2004

Page 602, Exercise 99:

$$\int \frac{1-\cos 2x}{1+\cos 2x}\,dx = \int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C.$$

Page 602, Exercise 101:

$$\int \frac{\cos x}{\sin^3 x - \sin x}\,dx = -\int \frac{\cos x}{(\sin x)(1-\sin^2 x)}\,dx = -\int \frac{\cos x}{(\sin x)(\cos^2 x)}\,dx$$

$$= -\int \frac{2}{\sin 2x}\,dx = -2\int \csc 2x\,dx = \ln|\csc 2x + \cot 2x| + C.$$

Page 603, Exercise 1:

Let $u = (\sin^{-1} x)^2$, and apply by parts to obtain

$$\begin{aligned}
\int \left(\sin^{-1} x\right)^2\,dx &= x\left(\sin^{-1} x\right)^2 - \int \frac{2x\sin^{-1} x}{\sqrt{1-x^2}}\,dx \\
&\quad \text{Now apply by parts in the second integral with } u = \sin^{-1} x, \\
&= x\left(\sin^{-1} x\right)^2 - \left[-2(\sin^{-1} x)\sqrt{1-x^2} + \int 2\,dx\right] \\
&= x\left(\sin^{-1} x\right)^2 + 2(\sin^{-1} x)\sqrt{1-x^2} - 2x + C.
\end{aligned}$$

Page 603, Exercise 3:

Let $u = \sin^{-1} x$, and apply by parts to obtain

$$\begin{aligned}
\int x\sin^{-1} x\,dx &= \frac{1}{2}x^2\sin^{-1} x - \int \frac{x^2}{2\sqrt{1-x^2}}\,dx \\
&\quad \text{Now in the second integral substitute } x = \sin\theta, \\
&= \frac{1}{2}x^2\sin^{-1} x - \frac{1}{2}\int \sin^2\theta\,d\theta \\
&= \frac{1}{2}x^2\sin^{-1} x - \frac{1}{2}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right) + C \\
&= \frac{1}{2}x^2\sin^{-1} x + \frac{\sin\theta\cos\theta - \theta}{4} + C \\
&= \frac{1}{2}x^2\sin^{-1} x + \frac{x\sqrt{1-x^2} - \sin^{-1} x}{4} + C.
\end{aligned}$$

Page 603, Exercise 9:

$$
\begin{aligned}
\int \frac{1}{x^4+1}\,dx &= \int \frac{1}{(x^2+1)^2-4x^2}\,dx \\
&= \int \frac{1}{(x^2+2x+2)(x^2-2x+2)}\,dx \\
&= \frac{1}{16}\int \left[\frac{2x+2}{x^2+2x+2} + \frac{2}{(x+1)^2+1} - \frac{2x-2}{x^2-2x+2} + \frac{2}{(x-1)^2+1}\right]\,dx \\
&= \frac{1}{16}\ln\left|\frac{x^2+2x+2}{x^2-2x+2}\right| + \frac{1}{8}\left[\tan^{-1}(x+1)+\tan^{-1}(x-1)\right] + C.
\end{aligned}
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