Quiz # 3
Math 101-Section 09 Calculus I
6 October 2015, Tuesday

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YOUR NAME:

In this quiz you can use only pencils and erasers.

Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) We have a function defined as

$$f(x) = \begin{cases} \frac{1-\cos x}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(i) Calculate $f'(0)$.

(ii) Write an equation of the tangent line to the curve $y = f(x)$ at $x = 0$.

(iii) Calculate $f'(\pi)$.

(iv) Write an equation of the tangent line to the curve $y = f(x)$ at $x = \pi$.

: Grading is 25 points for each question.

Answer:

$$f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x} = \lim_{x\to 0} \frac{\frac{1-\cos x}{x} - 0}{x} = \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}.$$

An equation of the tangent line at $x = 0$ is given as $y = f'(0)(x - 0) + f(0)$, so it is written as

$$y = (1/2)x.$$

On the other hand we have for $x \neq 0$,

$$f'(x) = \left(\frac{1-\cos x}{x}\right)' = \frac{(\sin x)(x) - (1-\cos x)(1)}{x^2}.$$

Hence we have $f'(\pi) = -\frac{2}{\pi^2}$. And an equation for the tangent line is given by

$$y = -\frac{2}{\pi^2}(x - \pi) + \frac{2}{\pi},$$

where we used $f(\pi) = 2/\pi$.