Bilkent University
Department of Mathematics

Instructor: Ali Sinan Sertöz

Quiz # 7

Math 101-Section 09 Calculus I
12 November 2015, Thursday

YOUR NAME:

In this quiz you can use only pencils and erasers.

Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) Let A be the area under the graph of an increasing continuous function f from a to b, and let L_n and R_n be the Riemann sum approximations to A with n equal subintervals using left and right endpoints, respectively.

(a) Show that $R_n - L_n = \dfrac{b-a}{n}[f(b) - f(a)]$.

(b) Show that $R_n - A < \dfrac{b-a}{n}[f(b) - f(a)]$.

(c) Now set $f(x) = \sin x^2$, $a = 0$ and $b = 1.2$. Take $\sin(1.44) = 0.9914$. Show that R_{1190} approximates A with an error strictly less than 0.001.

Answer:

(a) Here $\Delta x = (b-a)/n$, and

$$\begin{aligned} R_n &= f(a+\Delta x)\Delta x + f(a+2\Delta x)\Delta x + \cdots + f(a+k\Delta x)\Delta x + \cdots + f(a+(n-1)\Delta x)\Delta x + f(b)\Delta x, \\ L_n &= f(a)\Delta x + f(a+\Delta x)\Delta x + \cdots + f(a+k\Delta x)\Delta x + \cdots + f(a+(n-1)\Delta x)\Delta x. \end{aligned}$$

Then we see that $R_n - L_n = \dfrac{b-a}{n}[f(b) - f(a)]$.

(b) Since $L_n < A$, we have $R_n - A < R_n - L_n$. Now use part (a).

(c) Here $\dfrac{b-a}{n} = \dfrac{1.2}{n}$ and $f(b) - f(a) = \sin 1.44 = 0.9914$. We impose the condition

$$R_n - A < \frac{b-a}{n}[f(b) - f(a)] = \frac{1.2}{n}0.9914 < 0.001.$$

This gives $n > 1.2 \times 991.4 = 1189.68$. So taking $n = 1190$ gives the required precision.

($R_{1190} = 0.496615$, so $0.4956 < A < 0.4976$. Hence $A = 0.49$ correct to two decimal places.)