



**Quiz # 9**  
Math 101-Section 09 Calculus I  
26 November 2015, Thursday



Bilkent University  
Department of Mathematics

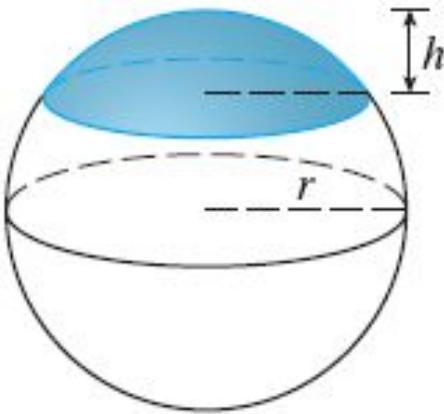
Instructor: Ali Sinan Sertöz

YOUR NAME:

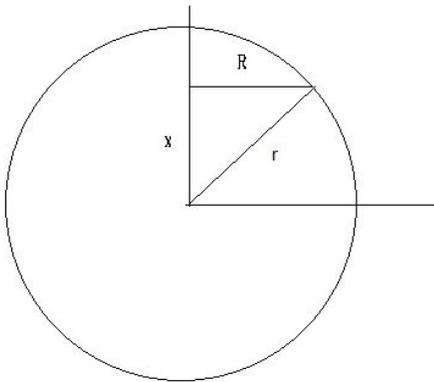
**In this quiz you can use only pencils and erasers.**

*Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.*

**Q-1)** Find the volume of the cap of height  $h$  cut off a sphere of radius  $r$ .



**Solution:**



If we cut the cap by a plane parallel to the equator of the sphere and  $x$  distance away from it, we obtain a circle whose radius is  $R$  as given in the above figure. Here  $R^2 = r^2 - x^2$ . The area of this circle is  $\pi R^2$ . Since the thickness of the cap is  $h$ , in the figure  $x$  runs from  $r - h$  to  $r$ .

Hence the volume is given by

$$V = \pi \int_{r-h}^r R^2 dx = \pi \int_{r-h}^r (r^2 - x^2) dx = \pi \left( r^2 x - \frac{1}{3} x^3 \Big|_{r-h}^r \right) = \pi h^2 \left( r - \frac{1}{3} h \right).$$