



Quiz # 10
Math 101-Section 09 Calculus I
8 December 2015, Thursday



Bilkent University
Department of Mathematics

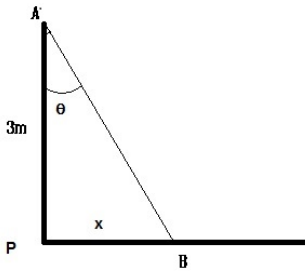
Instructor: Ali Sinan Sertöz

YOUR NAME:

In this quiz you can use only pencils and erasers.

Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) A small lighthouse is located on a small rock, 3m away from the nearest point P on a straight shoreline, and its light makes four revolutions per minute. How fast, in m/min , the beam of light moving along the shoreline when it is 1m away from P ?



Here θ and x are functions of time t .

Solution:

The light of the lighthouse is at the point A . It rotates 4 full revolutions per minute. Each revolution is 2π radians. So $\theta(t)' = 4 \text{ rev}/\text{min} = 8\pi \text{ rad}/\text{min}$.

The tip of the light touches the shoreline at a typical point say B . The point B is moving at an increasing rate away from P as its distance from P increases. The distance between A and P is $x(t)$. Observe that this is a function of time.

Now we put all these into one equation.

$$\tan \theta(t) = \frac{x(t)}{3}.$$

Differentiating both sides with respect to time we get $\sec^2 \theta(t) \cdot \theta(t)' = x(t)'/3$. Note that $\sec^2 \theta(t) = (9 + x^2(t))/3$. So $x'(t) = (8\pi(9 + x^2(t)))/3 \text{ m}/\text{min}$. Putting $x(t) = 1$ we get

$$x' = \frac{80\pi}{3} \text{ m}/\text{min}.$$