Bilkent University
Department of Mathematics

Quiz # 11
Math 101-Section 09 Calculus I
15 December 2015, Tuesday

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YOUR NAME:

In this quiz you can use only pencils and erasers.

Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) Evaluate $\displaystyle\int x \arcsin x \, dx$.

Hint: $(\arcsin x)' = 1/\sqrt{1-x^2}$.

Solution:

First use integration by parts. Let $u = \arcsin x$ and $dv = x\,dx$. Then $du = \dfrac{dx}{\sqrt{1-x^2}}$ and $v = \dfrac{x^2}{2}$. Thus we get

$$\int x \arcsin x \, dx = \frac{1}{2}\, x^2 \arcsin x - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}}\, dx.$$

Now we attack the second integral. Here make the substitution $x = \sin\theta$. We then get

$$\int \frac{x^2}{\sqrt{1-x^2}}\, dx = \int \sin^2\theta \, d\theta = \int \frac{1-\cos 2\theta}{2}\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C = \frac{1}{2}\theta - \frac{1}{2}\sin\theta\cos\theta + C.$$

Using the right triangle which gives $\sin\theta = x$ we find that $\cos\theta = \sqrt{1-x^2}$. This gives

$$\int \frac{x^2}{\sqrt{1-x^2}}\, dx = \frac{1}{2}\arcsin x - \frac{1}{2} x\sqrt{1-x^2} + C.$$

Combining this with the first result above, we find

$$\int x \arcsin x \, dx = \frac{1}{2}\, x^2 \arcsin x - \frac{1}{4}\arcsin x + \frac{1}{4} x\sqrt{1-x^2} + C.$$