Quiz # 2
Math 101-Section **011** Calculus I
13 October 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: ....

Student ID: ....   Your Department: ....

Show your work in detail. Correct answers without justification are never graded.

Q-1) Assume that the constants a and b are so chosen that the function

$$f(x) = \begin{cases} 2x^2 + ax - 7 & \text{if } x \geq 0 \\ 7x^4 - 8x^3 + 9x + b & \text{if } x < 0 \end{cases}$$

is differentiable at $x = 0$. Find $f(0)$ and $f'(0)$. (5+5 points)

Answer: If the function is differentiable at $x = 0$, then it must be continuous at $x = 0$. We must then have

$$f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x).$$

We have

$$\lim_{x \to 0^-} f(x) = b, \quad \text{and} \quad \lim_{x \to 0^+} f(x) = -7,$$

so

$$b = -7 = f(0).$$

Next we calculate the right and left limits of $\dfrac{f(x) - f(0)}{x}$ as x approaches zero.

$$f'(0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x} = \lim_{x \to 0^-} \frac{7x^4 - 8x^3 + 9x}{x} = \lim_{x \to 0^-} (7x^3 - 8x^2 + 9) = 9,$$

and

$$f'(0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x} = \lim_{x \to 0^+} \frac{2x^2 + ax}{x} = \lim_{x \to 0^+} (2x + a) = a.$$

Hence

$$a = 9 = f'(0).$$

Here is the graph of $y = f(x)$: