Bilkent University

Your Name: ....

Student ID: .... Your Department: ....

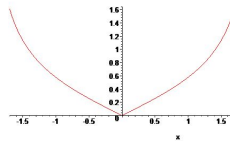*Show your work in detail. Correct answers without justification are never graded.*

Q-1) Let $f(x) = \dfrac{|x|}{\sqrt{4 - x^2}}$.

1. Find the domain of f. 2 points

2. Write an equation for the tangent line to the curve $y = f(x)$ at $x = 1$, if it exists. 4 points

3. Write an equation for the tangent line to the curve $y = f(x)$ at $x = 0$, if it exists. 4 points

Here is the graph of $y = f(x)$

Answer: The domain of f is all x with $-2 < x < 2$.
By direct computation we find that the derivative of f is

$$f'(x) = \begin{cases} \dfrac{4}{(4 - x^2)^{3/2}}, & \text{if } x > 0, \\[4mm] \dfrac{-4}{(4 - x^2)^{3/2}}, & \text{if } x < 0. \end{cases}$$

We also note that

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x} = -\frac{1}{2}, \quad \text{and} \quad \lim_{x \to 0^+} \frac{f(x) - f(0)}{x} = \frac{1}{2}.$$

This shows that $f'(0)$ does not exist, so there is no tangent line at $x = 0$. However $f'(1) = 4/(3\sqrt{3})$, with $f(1) = 1/\sqrt{3}$, so an equation for the tangent line at $x = 1$ is

$$y - \frac{1}{\sqrt{3}} = \frac{4}{3\sqrt{3}}(x - 1).$$