Bilkent University

Your Name: ....

Student ID: ....   Your Department: ....

Show your work in detail. Correct answers without justification are never graded.
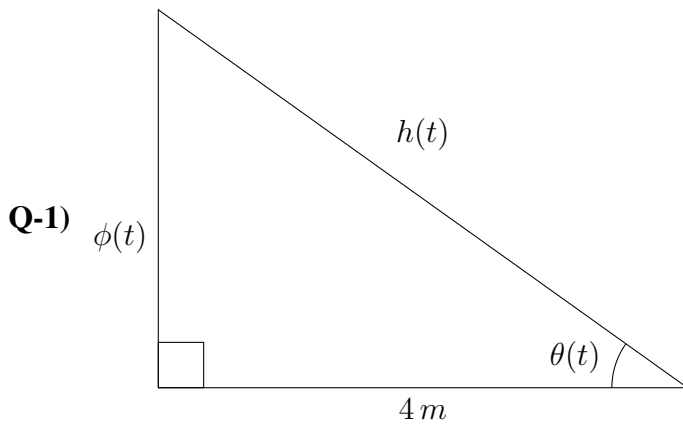
Q-1) We have $\phi(t) = t^3 + t^2 - t - 8$.
Find $\theta'(2)$.

Answer:

We have $\tan \theta(t) = \phi(t)/4$. Taking derivatives of both sides we get

$$(\sec^2 \theta(t))\, \theta'(t) = (3t^2 + 2t - 1)/4,$$

which gives

$$\theta'(t) = \cos^2 \theta(t)\, (3t^2 + 2t - 1)/4.$$

Putting $t = 2$, we get

$$\theta'(2) = \cos^2 \theta(2)(15/4).$$

To find $\cos \theta(2)$, we notice that

$$\phi(2) = 2 \quad \text{and} \quad h(2) = 2\sqrt{5}, \quad \text{hence} \quad \cos \theta(2) = \frac{2}{\sqrt{5}}.$$

Finally putting these back in the formula for $\theta'(2)$ we find

$$\theta'(2) = 3.$$

As an alternative solution we can write

$$\theta(t) = \arctan \frac{\theta(t)}{4}, \quad \text{and} \quad \theta'(t) = \frac{\theta'(t)/4}{(\theta(t)/4)^2 + 1}.$$

Now putting in $t = 2$, we get the result.