Quiz # 8
Math 101-Section **011** Calculus I
1 December 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

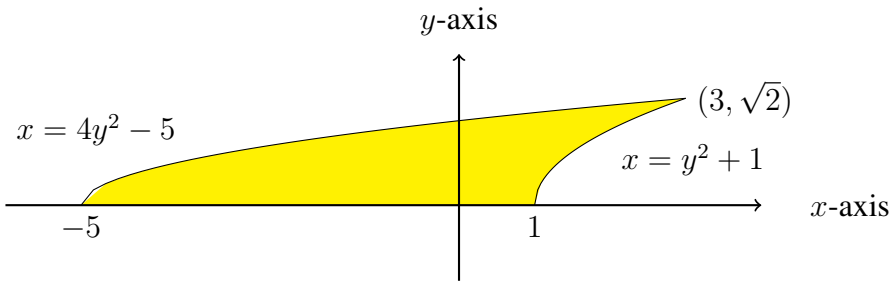
Your Name: .....

Student ID: ..... Your Department: .....

Show your work in detail. Correct answers without justification are never graded.

Q-1) Let D be the region bounded by the curves $y = 0$, $x = 4y^2 - 5$ and $x = y^2 + 1$. Find the volume of the solid obtained by revolving the region D around the x-axis. (10 points)

Answer:



For x between -5 and 3, a point on the curve $x = 4y^2 - 5$ is rotated around the x-axis to obtain a solid. From this solid we subtract the solid obtained by rotating a point on the curve $x = y^2 + 1$ for x between 1 and 3.

$$\begin{aligned} V &= \pi \int_{-5}^{3} y^2\,dx - \pi \int_{1}^{3} y^2\,dx \\ &= \pi \int_{-5}^{3} \frac{x+5}{4}\,dx - \pi \int_{1}^{3} (x-1)\,dx \\ &= \pi \left(\frac{1}{8}x^2 + \frac{5}{4}x \Big|_{-5}^{3} \right) - \pi \left(\frac{1}{2}x - x \Big|_{1}^{3} \right) \\ &= 8\pi - 2\pi = 6\pi. \end{aligned}$$

We can also use cylindrical shells. A typical point of height y on the curve $x = 4y^2 - 5$ is of the form $(4y^2 - 5, y)$. A typical point of height y on the curve $x = y^2 + 1$ is of the form $(y^2 + 1, y)$. The horizontal distance between them is $h = (y^2 + 1) - (4y^2 - 5)$. Then the volume is calculated as

$$V = 2\pi \int_0^{\sqrt{2}} y\,h\,dy = 2\pi \int_0^{\sqrt{2}} y\,(6 - 3y^2)\,dy = 2\pi \left(3y^2 - \frac{3}{4}y^4 \Big|_0^{\sqrt{2}} \right) = 6\pi.$$