



Quiz # 10  
 Math 101-Section 011 Calculus I  
 22 December 2016, Thursday  
 Instructor: Ali Sinan Sertöz  
**Solution Key**

10
----

Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

*This is going to count as an attendance quiz, so anyone who handed in a paper with his/her name correctly gets a full grade for this quiz.*

**Q-1)** Evaluate the indefinite integral  $\int \frac{x^4 + 2x^2 + x + 2}{(x + 1)(x^2 + 1)^2} dx$ .

(10 points)

**Answer:** First we write the partial fraction decomposition of the integrant.

$$\frac{x^4 + 2x^2 + x + 2}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$$

Multiplying both sides by  $x + 1$  and putting  $x = -1$  we get

$$A = 1.$$

Taking  $\frac{1}{x + 1}$  to the left hand side and simplifying we find that

$$\frac{x^4 + 2x^2 + x + 2}{(x + 1)(x^2 + 1)^2} - \frac{1}{x + 1} = \frac{1}{(x^2 + 1)^2}$$

This gives us

$$\frac{1}{(x^2 + 1)^2} = \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$$

From here we see that

$$B = C = D = 0, \text{ and } E = 1.$$

Hence we have

$$\frac{x^4 + 2x^2 + x + 2}{(x + 1)(x^2 + 1)^2} = \frac{1}{x + 1} + \frac{1}{(x^2 + 1)^2},$$

and

$$\int \frac{x^4 + 2x^2 + x + 2}{(x + 1)(x^2 + 1)^2} dx = \ln|x + 1| + \int \frac{dx}{(x^2 + 1)^2}$$

To evaluate the last integral we use integration by parts.

$$\begin{aligned} \int \frac{dx}{x^2 + 1} &= \frac{x}{x^2 + 1} + 2 \int \frac{x^2 dx}{(x^2 + 1)^2}, \quad (u = \frac{1}{x^2 + 1}, \quad dv = dx) \\ &= \frac{x}{x^2 + 1} + 2 \int \frac{dx}{x^2 + 1} - 2 \int \frac{dx}{(x^2 + 1)^2}, \quad (x^2 = (x^2 + 1) - 1) \end{aligned}$$

Now solving for  $\int \frac{dx}{(x^2 + 1)^2}$ , we find

$$\int \frac{dx}{(x^2 + 1)^2} = \frac{1}{2} \frac{x}{x^2 + 1} + \frac{1}{2} \int \frac{dx}{x^2 + 1} = \frac{1}{2} \frac{x}{x^2 + 1} + \frac{1}{2} \arctan x + C.$$

Putting these together we finally have

$$\int \frac{x^4 + 2x^2 + x + 2}{(x + 1)(x^2 + 1)^2} dx = \ln|x + 1| + \frac{1}{2} \frac{x}{x^2 + 1} + \frac{1}{2} \arctan x + C.$$