Quiz # 3
Math 101-Section **01** Calculus I
13 October 2017, Friday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: ....

Your Student ID: ....

Q-1) Find b such that the line $y = b - \sqrt{3}\,x$ is tangent to the ellipse $x^2 + 7y^2 = 616$ at a point in the first quadrant. (10 points)

Solution:
First find y' by implicit differentiation; differentiate both sides of $x^2 + 7y^2 = 616$ with respect to x to obtain $2x + 14y\,y' = 0$. Solving for y' we get $y' = -\dfrac{1}{7}\dfrac{x}{y}$, when $y \neq 0$. This is the slope of the tangent line $y = b - \sqrt{3}\,x$. Therefore $y' = -\dfrac{1}{7}\dfrac{x}{y} = -\sqrt{3}$. This gives $x = 7\sqrt{3}\,y$. Putting this back into the equation $x^2 + 7y^2 = 616$ and solving for y gives $y^2 = 4$. Since the tangency point is required to be in the first quadrant, we must have $y \geq 0$. So we get $y = 2$. This gives in return $x = 14\sqrt{3}$. Thus the point of tangency is $(14\sqrt{3}, 2)$. Now finally putting these values of x and y into the equation of the line and solving for b gives $b = 44$.

An alternate solution uses the parametrization of the given ellipse as $x = \sqrt{616}\sin\theta$ and $y = \sqrt{616/7}\sin\theta$. Putting these into the equation of y' as above we find that $\cot\theta = \sqrt{21}$. From here we find the values of $\sin\theta$ and $\cos\theta$ and obtain the coordinates of the tangency point. The rest is as above.