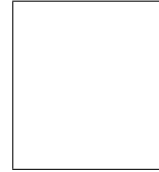




Quiz # 3
Math 101-Section 01 Calculus I
13 October 2017, Friday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

Your Student ID:

Q-1) Find b such that the line $y = b - \sqrt{3}x$ is tangent to the ellipse $x^2 + 7y^2 = 616$ at a point in the first quadrant. (10 points)

Solution:

First find y' by implicit differentiation; differentiate both sides of $x^2 + 7y^2 = 616$ with respect to x to obtain $2x + 14yy' = 0$. Solving for y' we get $y' = -\frac{1}{7} \frac{x}{y}$, when $y \neq 0$. This is the slope of the tangent line $y = b - \sqrt{3}x$. Therefore $y' = -\frac{1}{7} \frac{x}{y} = -\sqrt{3}$. This gives $x = 7\sqrt{3}y$. Putting this back into the equation $x^2 + 7y^2 = 616$ and solving for y gives $y^2 = 4$. Since the tangency point is required to be in the first quadrant, we must have $y \geq 0$. So we get $y = 2$. This gives in return $x = 14\sqrt{3}$. Thus the point of tangency is $(14\sqrt{3}, 2)$. Now finally putting these values of x and y into the equation of the line and solving for b gives $b = 44$.

An alternate solution uses the parametrization of the given ellipse as $x = \sqrt{616} \sin \theta$ and $y = \sqrt{616/7} \cos \theta$. Putting these into the equation of y' as above we find that $\cot \theta = \sqrt{21}$. From here we find the values of $\sin \theta$ and $\cos \theta$ and obtain the coordinates of the tangency point. The rest is as above.