Quiz # 9
Math 101-Section **01** Calculus I
8 December 2017, Friday
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Solution Key

Bilkent University

Your Name: ....

Your Student ID: ....

Q-1)

(a) Evaluate $\displaystyle\lim_{x\to 0}\frac{\tan x-\sin x}{x-x\cos x}$, (b) Evaluate $\displaystyle\int_1^e x\ln x^{2017}\,dx$. (5+5 points)

Solution:

(a) Apply L'Hospital's rule three times:

$$\begin{aligned}
\lim_{x\to 0}\frac{\tan x-\sin x}{x-x\cos x} &= \lim_{x\to 0}\frac{\sec^2 x-\cos x}{1-\cos x+x\sin x}\\
&= \lim_{x\to 0}\frac{2\sec^2 x\tan x+\sin x}{2\sin x+x\cos x}\\
&= \lim_{x\to 0}\frac{4\sec^2 x\tan^2 x+2\sec^4 x+\cos x}{3\cos x-x\sin x}=\frac{3}{3}=1.
\end{aligned}$$

(b) First note that $\ln x^{2017}=2017\ln x$, so we need to find an antiderivative for $x\ln x$. An easy way for this is to notice that

$$\frac{d}{dx}\left(x^2\ln x\right)=2x\ln x+x,$$

so we need something to cancel out the extra x. This is easy;

$$\frac{d}{dx}\left(x^2\ln x-\frac{1}{2}x^2\right)=2x\ln x-x+x=2x\ln x.$$

We now divide by 2 to get rid of the extra 2, and find the required antiderivative as

$$\int x\ln x\,dx=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+C.$$

If you are familiar with integration by parts techniques, then start with $u=\ln x$ and $dv=dx$ to get

$$\begin{aligned}
\int_1^e x\ln x^{2017}\,dx &= 2017\int_1^e x\ln x\,dx\\
&= 2017\left[\left(\frac{1}{2}x^2\ln x\Big|_1^e\right)-\frac{1}{2}\int_1^e x\,dx\right]\\
&= 2017\left(\frac{1}{2}x^2\ln x-\frac{1}{4}x^2\Big|_1^e\right)\\
&= 2017\left(\frac{e^2+1}{4}\right).
\end{aligned}$$