Quiz # 10
Math 101-Section **01** Calculus I
15 December 2017, Friday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: ....

Your Student ID: ....

Q-1)

(a) Evaluate $\displaystyle\lim_{x\to 0}\frac{\sec^2 x\tan^2 x}{(e^x+1)(x^2-1)(1-\cos x)}$, (b) Evaluate $\displaystyle\int \cos^4 x\,dx$. (5+5 points)

Solution:

(a) Apply L'Hospital's rule but first separate the components which do not cause indeterminacy:

$$\begin{aligned}\lim_{x\to 0}\frac{\sec^2 x\tan^2 x}{(e^x+1)(x^2-1)(1-\cos x)} &= \lim_{x\to 0}\frac{\sec^2 x}{(\cos^2 x)(e^x+1)(x^2-1)}\lim_{x\to 0}\frac{\sin^2 x}{(1-\cos x)}\\ &= -\frac{1}{2}\lim_{x\to 0}\frac{2\sin x\cos x}{\sin x}\\ &= -\frac{1}{2}\times 2=-1.\end{aligned}$$

(b) First use by-parts with $u=\cos^3 x$ and $dv=\cos x\,dx$.

$$\int \cos^4 x\,dx = \sin x\cos^3 x + 3\int \cos^2 x\sin^2 x\,dx.$$

Next treat the last integral

$$\int \cos^2 x\sin^2 x\,dx = \int \cos^2 x(1-\cos^2 x)\,dx = \int \cos^2 x\,dx - \int \cos^4 x\,dx.$$

But $\cos^2 x$ is easily integrated.

$$\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}x+\frac{1}{4}\sin 2x + C.$$

Putting these together and solving for $\int \cos^4 x\,dx$ we get

$$\int \cos^4 x\,dx = \frac{1}{4}\sin x\cos^3 x + \frac{3}{8}x + \frac{3}{16}\sin 2x + C.$$