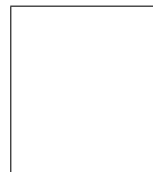




Quiz # 10
 Math 101-Section 01 Calculus I
 15 December 2017, Friday
 Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

Your Student ID:

Q-1)

(a) Evaluate $\lim_{x \rightarrow 0} \frac{\sec^2 x \tan^2 x}{(e^x + 1)(x^2 - 1)(1 - \cos x)}$, (b) Evaluate $\int \cos^4 x \, dx$. (5+5 points)

Solution:

(a) Apply L'Hospital's rule but first separate the components which do not cause indeterminacy:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sec^2 x \tan^2 x}{(e^x + 1)(x^2 - 1)(1 - \cos x)} &= \lim_{x \rightarrow 0} \frac{\sec^2 x}{(\cos^2 x)(e^x + 1)(x^2 - 1)} \lim_{x \rightarrow 0} \frac{\sin^2 x}{(1 - \cos x)} \\ &= -\frac{1}{2} \lim_{x \rightarrow 0} \frac{2 \sin x \cos x}{\sin x} \\ &= -\frac{1}{2} \times 2 = -1. \end{aligned}$$

(b) First use by-parts with $u = \cos^3 x$ and $dv = \cos x \, dx$.

$$\int \cos^4 x \, dx = \sin x \cos^3 x + 3 \int \cos^2 x \sin^2 x \, dx.$$

Next treat the last integral

$$\int \cos^2 x \sin^2 x \, dx = \int \cos^2 x (1 - \cos^2 x) \, dx = \int \cos^2 x \, dx - \int \cos^4 x \, dx.$$

But $\cos^2 x$ is easily integrated.

$$\int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} \, dx = \frac{1}{2} x + \frac{1}{4} \sin 2x + C.$$

Putting these together and solving for $\int \cos^4 x \, dx$ we get

$$\int \cos^4 x \, dx = \frac{1}{4} \sin x \cos^3 x + \frac{3}{8} x + \frac{3}{16} \sin 2x + C.$$