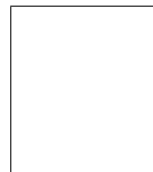




Quiz # 2  
 Math 101-Section 06 Calculus I  
 15 February, 2018, Thursday  
 Instructor: Ali Sinan Sertöz  
**Solution Key**



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**Q-1)** Let  $f(x) = \frac{x}{1+x^2}$ .

- (i) Write an equation for the tangent line to  $y = f(x)$  at  $x = t$
- (ii) Let  $A(t)$  denote the area of the triangle formed by the coordinate axes and the above tangent line. Find  $\lim_{t \rightarrow \infty} A(t)$ .

**Answer:** If  $f(x) = \frac{x}{1+x^2}$ , then  $f'(x) = \frac{1-x^2}{(1+x^2)^2}$ , and at the point  $x = t$ , the slope of the tangent line is  $f'(t) = \frac{1-t^2}{(1+t^2)^2}$ . Then an equation for the tangent line at the point  $(t, \frac{t}{(1+t^2)})$  is

$$y = \frac{1-t^2}{(1+t^2)^2}(x-t) + \frac{t}{(1+t^2)}.$$

When  $x = 0$ , we find that the  $y$ -intercept is  $y_0$ , and when  $y = 0$ , we find that the  $x$ -intercept is  $x_0$ , where

$$x_0 = \frac{2t^3}{t^2-1}, \quad y_0 = \frac{2t^3}{(1+t^2)^2}.$$

Thus the area of the above mentioned triangle is

$$A(t) = \frac{1}{2} x_0 y_0 = \frac{2t^6}{(t^2-1)(1+t^2)^2} = \frac{2t^6}{t^6+t^4-t^2-1}.$$

Finally, we evaluate the limit of this as  $t$  goes to infinity, to find

$$\lim_{t \rightarrow \infty} A(t) = 2.$$