Quiz # 2
Math 101-Section **06** Calculus I
15 February, 2018, Thursday
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Solution Key

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Q-1) Let $f(x) = \dfrac{x}{1+x^2}$.

(i) Write an equation for the tangent line to $y = f(x)$ at $x = t$

(ii) Let $A(t)$ denote the area of the triangle formed by the coordinate axes and the above tangent line. Find $\lim\limits_{t\to\infty} A(t)$.

Answer: If $f(x) = \dfrac{x}{1+x^2}$, then $f'(x) = \dfrac{1-x^2}{(1+x^2)^2}$, and at the point $x = t$, the slope of the tangent line is $f'(t) = \dfrac{1-t^2}{(1+t^2)^2}$. Then an equation for the tangent line at the point $(t, \dfrac{t}{(1+t^2)})$ is

$$y = \frac{1-t^2}{(1+t^2)^2}(x-t) + \frac{t}{(1+t^2)}.$$

When $x = 0$, we find that the y-intercept is y_0, and when $y = 0$, we find that the x-intercept is x_0, where

$$x_0 = \frac{2t^3}{t^2-1}, \quad y_0 = \frac{2t^3}{(1+t^2)^2}.$$

Thus the area of the above mentioned triangle is

$$A(t) = \frac{1}{2}\, x_0\, y_0 = \frac{2t^6}{(t^2-1)(1+t^2)^2} = \frac{2t^6}{t^6+t^4-t^2-1}.$$

Finally, we evaluate the limit of this as t goes to infinity, to find

$$\lim_{t\to\infty} A(t) = 2.$$