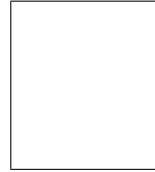




Quiz # 3
 Math 101-Section 01 Calculus I
 23 February, 2018, Friday
 Instructor: Ali Sinan Sertöz
Solution Key



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Name:

Department:

Student ID:

Q-1) Let $f(x) = \frac{2x^2 + x + 2}{2x^4 + x^2 + 1}$, and $g(x) = \frac{1 + \sin x}{2 + \cos x}$.

(i) Calculate $(f \circ g)'(\frac{\pi}{2})$.

(ii) Calculate $(f \circ f)'(2)$.

Answer:

$$g'(x) = \frac{(\cos x)(2 + \cos x) - (1 + \sin x)(-\sin x)}{(2 + \cos x)^2}$$

$$g'(\frac{\pi}{2}) = \frac{1}{2}$$

$$f'(x) = \frac{(4x + 1)(2x^4 + x^2 + 1) - (2x^2 + x + 1)(8x^3 + 2x)}{(2x^4 + x^2 + 1)^2}$$

$$g(\frac{\pi}{2}) = 1$$

$$f'(1) = -\frac{15}{8}$$

$$(f \circ g)'(\frac{\pi}{2}) = f'(g(\frac{\pi}{2}))g'(\frac{\pi}{2}) = f'(1)g'(\frac{\pi}{2}) = -\frac{15}{8} \cdot \frac{1}{2} = -\frac{15}{16}.$$

$$f'(0) = 1$$

$$f(0) = 2$$

$$f'(2) = -\frac{483}{1369}$$

$$(f \circ f)'(0) = f'(f(0))f'(0) = f'(2)f'(0) = -\frac{483}{1369}.$$

There is a typo in the question: I wanted to ask $(f \circ f)'(0)$. All correct calculations, without simplification, will be granted full marks for $(f \circ f)'(2)$, whose calculation is as follows:

$$f(2) = \frac{12}{37}, \quad f'(\frac{12}{37}) = \frac{891784814545}{4463792847361}, \quad f'(2) = -\frac{483}{1369},$$

and

$$(f \circ f)'(2) = f'(\frac{12}{37})f'(2) = -\frac{314632626315}{4463792847361} \approx -0.07.$$