Quiz # 3
Math 101-Section **01** Calculus I
23 February, 2018, Friday
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Solution Key

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Q-1) Let $f(x) = \dfrac{2x^2+x+2}{2x^4+x^2+1}$, and $g(x) = \dfrac{1+\sin x}{2+\cos x}$.

(i) Calculate $(f \circ g)'\,(\frac{\pi}{2})$.

(ii) Calculate $(f \circ f)'\,(2)$.

Answer:

$$\begin{aligned}
g'(x) &= \frac{(\cos x)(2+\cos x) - (1+\sin x)(-\sin x)}{(2+\cos x)^2} \\
g'(\frac{\pi}{2}) &= \frac{1}{2} \\
f'(x) &= \frac{(4x+1)(2x^4+x^2+1) - (2x^2+x+1)(8x^3+2x)}{(2x^4+x^2+1)^2} \\
g(\frac{\pi}{2}) &= 1 \\
f'(1) &= -\frac{15}{8} \\
(f \circ g)'\,(\frac{\pi}{2}) &= f'(g(\frac{\pi}{2}))\,g'(\frac{\pi}{2}) = f'(1)\,g'(\frac{\pi}{2}) = -\frac{15}{8}\,\frac{1}{2} = -\frac{15}{16}.
\end{aligned}$$

$$\begin{aligned}
f'(0) &= 1 \\
f(0) &= 2 \\
f'(2) &= -\frac{483}{1369} \\
(f \circ f)'\,(0) &= f'(f(0))\,f'(0) = f'(2)f'(0) = -\frac{483}{1369}.
\end{aligned}$$

There is a typo in the question: I wanted to ask $(f \circ f)'(0)$. All correct calculations, without simplification, will be granted full marks for $(f \circ f)'(2)$, whose calculation is as follows:

$$f(2) = \frac{12}{37}, \quad f'(\frac{12}{37}) = \frac{891784814545}{4463792847361}, \quad f'(2) = -\frac{483}{1369},$$

and

$$(f \circ f)'(2) = f'(\frac{12}{37})f'(2) = -\frac{314632626315}{4463792847361} \approx -0.07.$$