

Quiz # 5 Math 101-Section **01** Calculus I 9 March, 2018, Friday Instructor: Ali Sinan Sertöz

Solution Key

Bilkent University

	Name:
Department:	Student ID:

Q-1) Let
$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$
.

- (i) Calculate, and simplify f'(x).
- (ii) Calculate, and simplify f''(x).
- (iii) On what intervals is f concave upward? Concave downward?

Answer:

(i)
$$f'(x) = \frac{4x}{(x^2+1)^2}$$
.

(ii)
$$f''(x) = -4 \frac{3x^4 + 2x^2 - 1}{(x^2 + 1)^4} = -4 \frac{(3x^2 - 1)(x^2 + 1)}{(x^2 + 1)^4} = -4 \frac{3x^2 - 1}{(x^2 + 1)^3}.$$

(iii) f is concave down on $(-\infty, -\frac{1}{\sqrt{3}})$ and on $(\frac{1}{\sqrt{3}}, \infty)$.

And f is concave up on $(-\frac{1}{\sqrt{3}},(\frac{1}{\sqrt{3}}).$