Bilkent University

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Q-1) Let $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

    (i) Calculate, and simplify $f'(x)$.

    (ii) Calculate, and simplify $f''(x)$.

    (iii) On what intervals is f concave upward? Concave downward?

Answer:

(i)
$$f'(x) = \frac{4x}{(x^2 + 1)^2}.$$

(ii)
$$f''(x) = -4\,\frac{3x^4 + 2x^2 - 1}{(x^2 + 1)^4} = -4\,\frac{(3x^2 - 1)(x^2 + 1)}{(x^2 + 1)^4} = -4\,\frac{3x^2 - 1}{(x^2 + 1)^3}.$$

(iii)
f is concave down on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and on $\left(\frac{1}{\sqrt{3}}, \infty\right)$.

And f is concave up on $\left(-\frac{1}{\sqrt{3}}, \left(\frac{1}{\sqrt{3}}\right)\right)$.