Quiz # 10
Math 101-Section **06** Calculus I
19 April, 2018, Thursday
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Solution Key

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Q-1) Let $a > 1$ be any real number.

(i) Find and simplify the derivatives of

$$f(x) = \frac{a^{2x}}{2\ln a}, \quad \text{and} \quad g(x) = \frac{a^x}{(\ln a)^2}(x\ln a - 1).$$

(ii) Let R be the region bounded by the curves $x = 0$, $x = 1$, $y = 0$ and $y = a^x$. Find the volume of the solid obtained by revolving R around the x-axis.

(iii) Find the volume of the solid obtained by revolving R around the y-axis.

Answer:

(i) $f'(x) = a^{2x}$ and $g'(x) = xa^x$.

(ii)

$$V = \pi \int_0^1 (a^x)^2\, dx = \pi \left(f(x)\Big|_0^1 \right) = \pi\, \frac{a^2 - 1}{2\ln a}$$

(iii)

$$V = 2\pi \int_0^1 xa^x\, dx = 2\pi \left(g(x)\Big|_0^1 \right) = 2\pi\, \frac{a^x\left(-1 + x\ln(a)\right)}{(\ln(a))^2}.$$