Quiz # 5
Math 101-Section **09** Calculus I
9 November 2018, Friday
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Solution Key

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Q-1) let $f(x) = x^2 \sin \frac{1}{x}$ for $x \neq 0$, and $f(0) = 0$.

i. Calculate $f'(0)$ if it exists.

ii. Show that $y = x$ is an oblique asymptote for $y = f(x)$. (Hint: You may assume $\lim_{t \to 0} \frac{t - \sin t}{t^2} = 0$.)

Solution:

i. Note that

$$-x \leq x \sin \frac{1}{x} \leq x, \quad \text{for } x \neq 0,$$

so $\lim_{x \to 0} x \sin \frac{1}{x} = 0$ by the Sandwich Theorem. Hence

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x} = \lim_{x \to 0} x \sin \frac{1}{x} = 0.$$

ii.

$$\begin{aligned} \lim_{x \to \pm\infty} [f(x) - x] &= \lim_{x \to \pm\infty} [x^2 \sin \frac{1}{x} - x] \\ &= \lim_{t \to 0} [\frac{\sin t}{t^2} - \frac{1}{t}] \\ &= \lim_{t \to 0} \frac{\sin t - t}{t^2} \\ &= 0, \quad \text{from the given hint.} \end{aligned}$$

Hence $y = x$ is an asymptote. Here is the graph with $y = f(x)$ and $y = x$.