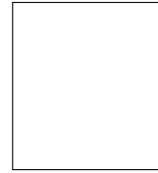




Quiz # 5
Math 101-Section 09 Calculus I
9 November 2018, Friday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Q-1) let $f(x) = x^2 \sin \frac{1}{x}$ for $x \neq 0$, and $f(0) = 0$.

i. Calculate $f'(0)$ if it exists.

ii. Show that $y = x$ is an oblique asymptote for $y = f(x)$. (Hint: You may assume $\lim_{t \rightarrow 0} \frac{t - \sin t}{t^2} = 0$.)

Solution:

i. Note that

$$-x \leq x \sin \frac{1}{x} \leq x, \text{ for } x \neq 0,$$

so $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$ by the Sandwich Theorem. Hence

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0.$$

ii.

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} [f(x) - x] &= \lim_{x \rightarrow \pm\infty} \left[x^2 \sin \frac{1}{x} - x \right] \\ &= \lim_{t \rightarrow 0} \left[\frac{\sin t}{t^2} - \frac{1}{t} \right] \\ &= \lim_{t \rightarrow 0} \frac{\sin t - t}{t^2} \\ &= 0, \text{ from the given hint.} \end{aligned}$$

Hence $y = x$ is an asymptote. Here is the graph with $y = f(x)$ and $y = x$.

