Quiz # 8
Math 101-Section **09** Calculus I
30 November 2018, Friday
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Solution Key

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Q-1) Calculate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n+k}{\sqrt{2n^4+2kn^3+k^2n^2}}$.

Solution:

Let

$$f(x)=\frac{x}{\sqrt{1+x^2}},\quad x_k=1+\frac{k}{n},\text{ and }\Delta x=\frac{1}{n}.$$

Then

$$\frac{n+k}{\sqrt{2n^4+2kn^3+k^2n^2}}=\frac{1+\frac{k}{n}}{\sqrt{1+(1+\frac{k}{n})^2}}\,\frac{1}{n}=f(x_k)\,\Delta x.$$

Note that f is defines on $[x_0,x_n]=[1,2]$.

Hence the given sum is the Riemann sum of $f(x)$ on $[1,2]$. It is then equal to the following integral.

$$\int_1^2\frac{x}{\sqrt{1+x^2}}\,dx=\left(\sqrt{1+x^2}\Big|_1^2\right)=\sqrt{5}-\sqrt{2}\approx 0.82.$$