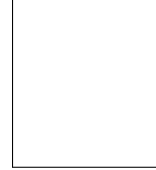




Quiz # 8
Math 101-Section 09 Calculus I
30 November 2018, Friday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Q-1) Calculate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n+k}{\sqrt{2n^4 + 2kn^3 + k^2n^2}}$.

Solution:

Let

$$f(x) = \frac{x}{\sqrt{1+x^2}}, \quad x_k = 1 + \frac{k}{n}, \quad \text{and} \quad \Delta x = \frac{1}{n}.$$

Then

$$\frac{n+k}{\sqrt{2n^4 + 2kn^3 + k^2n^2}} = \frac{1 + \frac{k}{n}}{\sqrt{1 + (1 + \frac{k}{n})^2}} \frac{1}{n} = f(x_k) \Delta x.$$

Note that f is defined on $[x_0, x_n] = [1, 2]$.

Hence the given sum is the Riemann sum of $f(x)$ on $[1, 2]$. It is then equal to the following integral.

$$\int_1^2 \frac{x}{\sqrt{1+x^2}} dx = \left(\sqrt{1+x^2} \Big|_1^2 \right) = \sqrt{5} - \sqrt{2} \approx 0.82.$$