Bilkent University

Quiz # 9
Math 101-Section **09** Calculus I
7 December 2018, Friday
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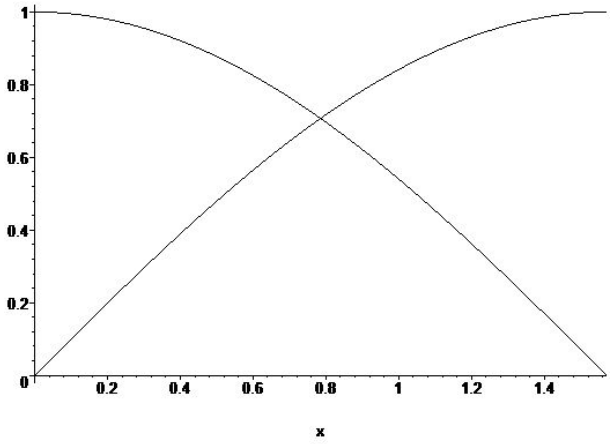
Solution Key

Q-1) Find the volume obtained by revolving around the x-axis the region between the curves $y = \sin x$ and $y = \cos x$ on $[0, \pi/2]$.

Solution:



We first find the intersection point by solving $\sin x = \cos x$, which gives $x = \pi/4$ on $[0, \pi/2]$.

The volume then becomes

$$\begin{aligned} V &= \pi \int_0^{\pi/4} [(\cos x)^2 - (\sin x)^2]\, dx + \pi \int_{\pi/4}^{\pi/2} [(\sin x)^2 - / \cos x)^2]\, dx \\ &= \pi \int_0^{\pi/4} \cos 2x\, dx - \pi \int_{\pi/4}^{\pi/2} \cos 2x\, dx \\ &= \pi \left(\frac{1}{2} \sin 2x \Big|_0^{\pi/4} \right) - \pi \left(\frac{1}{2} \sin 2x \Big|_{\pi/4}^{\pi/2} \right) \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi. \end{aligned}$$