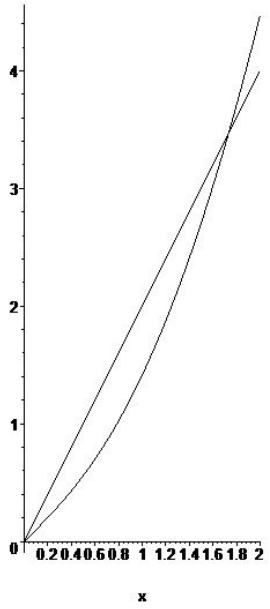
Bilkent University

Quiz # 9
Math 101-Section **13** Calculus I
6 December 2018, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Find the volume obtained by revolving around the x-axis the region between the curves $y = x\sqrt{1+x^2}$ and $y = 2x$ on $[0, 2]$.

Solution:



We first find the intersection point by solving $x\sqrt{1+x^2} = 2x$, which gives $x = 0$ and $x = \sqrt{3}$ on $[0, 2]$.

The volume then becomes

$$\begin{aligned}
V &= \pi \int_0^{\sqrt{3}} [(2x)^2 - (x\sqrt{1+x^2})^2]\, dx + \pi \int_{\sqrt{3}}^2 [(x\sqrt{1+x^2})^2 - (2x)^2]\, dx, \\
&= \pi \int_0^{\sqrt{3}} [3x^2 - x^4]\, dx + \pi \int_{\sqrt{3}}^2 [x^4 - 3x^2]\, dx, \\
&= \pi \left(x^3 - \frac{x^5}{5}\Big|_0^{\sqrt{3}} \right) + \pi \left(\frac{x^5}{5} - x^3 \Big|_{\sqrt{3}}^2 \right) \\
&= \pi \left(\frac{6\sqrt{3}}{5} \right) + \pi \left(\frac{6\sqrt{3}}{5} - \frac{8}{5} \right) \\
&= \frac{12\sqrt{3} - 8}{5}\, \pi \approx 2.56\, \pi \approx 8.03.
\end{aligned}$$