



Bilkent University

Quiz # 11
Math 101-Section 09 Calculus I
21 December 2018, Friday
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Solution Key



Q-1) Evaluate $\int_0^{\sqrt{3}/2} x^4 \sqrt{1-x^2} dx$. You may use the following identities.

$$\int \sin^2 \theta d\theta = \frac{\theta}{2} - \frac{1}{2} \sin \theta \cos \theta + C, \int \sin^4 \theta d\theta = \frac{3\theta}{8} - \frac{1}{4} \sin^3 \theta \cos \theta - \frac{3}{8} \sin \theta \cos \theta + C, \int \sin^6 \theta d\theta = \frac{5\theta}{16} - \frac{1}{6} \sin^5 \theta \cos \theta - \frac{5}{24} \sin^3 \theta \cos \theta - \frac{5}{16} \sin \theta \cos \theta + C$$

Solution:

We use trigonometric substitution $x = \sin \theta$. Then $dx = \cos \theta d\theta$, $x^4 \sqrt{1-x^2} = \sin^4 \theta \cos \theta$ and the integral becomes from $\theta = 0$ to $\theta = \pi/3$.

$$\begin{aligned} \int_0^{1/\sqrt{2}} x^4 \sqrt{1-x^2} dx &= \int_0^{\pi/3} \sin^4 \theta \cos^2 \theta d\theta = \int_0^{\pi/3} \sin^4 \theta (1 - \sin^2 \theta) d\theta \\ &= \int_0^{\pi/3} \sin^4 \theta d\theta - \int_0^{\pi/3} \sin^6 \theta d\theta \\ &= \left(\frac{3\theta}{8} - \frac{1}{4} \sin^3 \theta \cos \theta - \frac{3}{8} \sin \theta \cos \theta \Big|_0^{\pi/3} \right) \\ &\quad - \left(\frac{5\theta}{16} - \frac{1}{6} \sin^5 \theta \cos \theta - \frac{5}{24} \sin^3 \theta \cos \theta - \frac{5}{16} \sin \theta \cos \theta \Big|_0^{\pi/3} \right) \\ &= \frac{\pi}{48}. \end{aligned}$$