Quiz # 4
Math 101-Section **09** Calculus I
2 November 2018, Friday
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**Solution Key**

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**Q-1)** We have a water tank in the shape of an upside-down cone with radius $\pi$ cm and height 3 cm.Water is poured into the tank at the constant rate of 4 cm$^3$/sec. How fast is the water level rising when there is 27 cm$^3$ water in the tank?

**Solution:**

Let $h(t)$ be the height of water in the tank and $r(t)$ the radius of the surface of water at that time. We have

$$\frac{r(t)}{h(t)} = \frac{\pi}{3}, \quad \text{hence } r(t) = \frac{\pi}{3}\, h(t).$$

If $V(t)$ is the volume of water in the tank at time $t$, then

$$V'(t) = 4,$$

and

$$V(t) = \frac{\pi}{3}\, r(t)^2 h(t) = \frac{\pi^3}{27}\, h(t)^3,$$

from where we solve for $h(t)$ to find

$$h(t) = \frac{3}{\pi}\, V(t)^{1/3}.$$

Taking derivatives of both sides with respect to $t$, we find

$$h'(t) = \frac{1}{\pi}\, \frac{V'(t)}{V(t)^{2/3}}.$$

Since $V(t_0) = 27$ cm$^3$ and $V'(t_0) = 4$ cm$^3$/sec, we get

$$h(t_0) = \frac{4}{9\pi} \text{ cm/sec}.$$