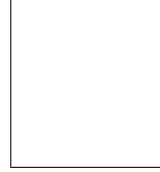




Quiz # 4
Math 101-Section 09 Calculus I
2 November 2018, Friday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Q-1) We have a water tank in the shape of an upside-down cone with radius π cm and height 3 cm. Water is poured into the tank at the constant rate of $4 \text{ cm}^3/\text{sec}$. How fast is the water level rising when there is 27 cm^3 water in the tank?

Solution:

Let $h(t)$ be the height of water in the tank and $r(t)$ the radius of the surface of water at that time. We have

$$\frac{r(t)}{h(t)} = \frac{\pi}{3}, \text{ hence } r(t) = \frac{\pi}{3} h(t).$$

If $V(t)$ is the volume of water in the tank at time t , then

$$V'(t) = 4,$$

and

$$V(t) = \frac{\pi}{3} r(t)^2 h(t) = \frac{\pi^3}{27} h(t)^3,$$

from where we solve for $h(t)$ to find

$$h(t) = \frac{3}{\pi} V(t)^{1/3}.$$

Taking derivatives of both sides with respect to t , we find

$$h'(t) = \frac{1}{\pi} \frac{V'(t)}{V(t)^{2/3}}.$$

Since $V(t_0) = 27 \text{ cm}^3$ and $V'(t_0) = 4 \text{ cm}^3/\text{sec}$, we get

$$h(t_0) = \frac{4}{9\pi} \text{ cm/sec}.$$