Bilkent University

Quiz # 02
Math 101-Section 08 Calculus I
10 October 2019, Thursday
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Solution Key

Q-1) A function f is defined as

$$f(x) = \begin{cases} x^2 + x\cos x + 2019 & x > 0, \\ \\ x^2 + ax + b & x \leq 0 \end{cases}$$

Where a and b are real numbers.

(i) Find all a and b such that f is continuous at $x = 0$. (4 points)

(ii) Find all a and b such that f is differentiable at $x = 0$. (6 points)

Solution:

(i) For continuity at $x = 0$ we need to have

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = f(0) = b.$$

We see that

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x^2 + x\cos x + 2019) = 2019, \text{ and } \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x^2 + ax + b) = b.$$

This imposes a condition only on b, so for continuity at the origin b must be 2019, and a can be arbitrary.

(ii) If f is going to be differentiable at the origin, it must be necessarily continuous. So we must have $b = 2019$ to begin with. We check if the existence of differentiability will impose a condition on a. For this we notice that if f is continuous at the origin then we must have

$$\lim_{x\to 0^+} \frac{f(x) - f(0)}{x} = \lim_{x\to 0^-} \frac{f(x) - f(0)}{x}.$$

We calculate these limits separately.

$$\lim_{x\to 0^+} \frac{f(x) - f(0)}{x} = \lim_{x\to 0^+} \frac{(x^2 + x\cos x + 2019) - (2019)}{x} = \lim_{x\to 0^+} (x + \cos x) = 1,$$

and

$$\lim_{x\to 0^-} \frac{f(x) - f(0)}{x} = \lim_{x\to 0^-} \frac{(x^2 + ax + 2019) - (2019)}{x} = \lim_{x\to 0^-} (x + a) = a.$$

This shows that for differentiability at the origin, besides having $b = 2019$, we must also have $a = 1$.