Bilkent University

Quiz # 04
Math 101-Section 08 Calculus I
24 October 2019, Thursday
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Solution Key

Q-1) Let $f(x,y) = \dfrac{1}{16}x^4y^4 - \dfrac{1}{2}x^2y^3 + \dfrac{1}{3}x^3y^2 + 2x.$

(i) Implicitly differentiate $f(x,y) = 0$ with respect to x, assuming that y is a differentiable function of x. (3 points)

(ii) Write an equation of the tangent line to the curve $f(x,y) = \dfrac{1}{3}$, at the point $(1,2)$ on the curve. (2 points)

(iii) Write an equation of the tangent line to the curve $f(x,y) = -\dfrac{23}{3}$, at the point $(-2,1)$ on the curve. (2 points)

(iv) Find the point where these two tangent lines intersect. (3 points)

Remark: It is true that $f(1,2) = \dfrac{1}{3}$ and $f(-2,1) = -\dfrac{23}{3}$. You need not check these facts in this exam.

Solution:

(i) $\dfrac{1}{4}x^3y^4 - xy^3 + x^2y^2 + 2 + \left(\dfrac{1}{4}x^4y^3 - \dfrac{3}{2}x^2y^2 + \dfrac{2}{3}x^3y\right)\, y' = 0.$

(ii) Let $g(x,y) = \dfrac{1}{4}x^3y^4 - xy^3 + x^2y^2 + 2 + \left(\dfrac{1}{4}x^4y^3 - \dfrac{3}{2}x^2y^2 + \dfrac{2}{3}x^3y\right)\, y'$. Differentiating both sides of $f(x,y) = \dfrac{1}{3}$ with respect to x assuming that y is a differentiable function of x around the point $(1,2)$ gives $g(x,y) = 0$. Solving for y' from $g(1,2) = 0$ we get $y' = \dfrac{3}{4}$. Hence an equation for the tangent line at that point is

$$y = L_1(x) \text{ where } L_1(x) = \frac{3}{4}(x-1) + 2.$$

(iii) Differentiating both sides of $f(x,y) = -\dfrac{23}{3}$ with respect to x assuming that y is a differentiable function of x around the point $(-2,1)$ gives $g(x,y) = 0$. Solving for y' from $g(-2,1) = 0$ we get $y' = \dfrac{9}{11}$. Hence an equation for the tangent line at that point is

$$y = L_2(x) \text{ where } L_2(x) = \frac{9}{11}(x+2) + 1.$$

(iv) Solving $L_1(x) = L_2(x)$, we find $x = -\dfrac{61}{3}$. Solving for y, either from $y = L_1(-\frac{61}{3})$ or from $y = L_2(-\frac{61}{3})$, we find $y = -14$. Hence these two tangent lines intersect at $\left(-\dfrac{61}{3}, -14\right)$.