Bilkent University

Quiz # 08
Math 101-Section 08 Calculus I
21 November 2019, Thursday
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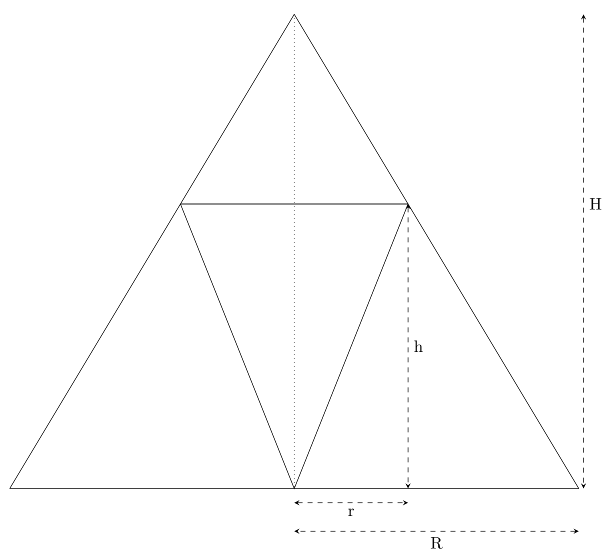
Solution Key

Q-1) Inside a right circular cone of base radius R and height H we insert an upside down right circular cone of base radius r and height h in such a manner that the bases of the cones are parallel. Find, in terms of R and H, the maximum volume that the inserted cone can attain.

Hint: The volume of the above big cone is $\frac{1}{3}\pi R^2 H$.

Solution:



From similar triangles we get $\dfrac{H-h}{r} = \dfrac{H}{R}$. From which we get $h = H - \dfrac{rH}{R}$.

We substitute this into the volume formula $V = \dfrac{\pi}{3} r^2 h$ for the inserted cone to obtain

$$V(r) = \frac{\pi H}{3}\left(r^2 - \frac{1}{R} r^3\right), \; 0 \le r \le R.$$

Taking derivative with respect to r we get

$$V'(r) = \frac{\pi H}{3}\left(2r - \frac{3}{R} r^2\right) = 0, \text{ which gives } r = 0 \text{ and } r = \frac{2}{3}R.$$

Evaluating $V(r)$ at the critical points and at the end points we get:

$$V(0) = 0, \quad V(\frac{2}{3}R) = \frac{4}{81}\pi R^2 H, \quad V(R) = 0.$$

Hence the maximum possible volume is $\dfrac{4}{81}\pi R^2 H$, or equivalently $\dfrac{4}{27}$ of the volume of the big cone.