



Bilkent University

Quiz # 10  
Math 101-Section 08 Calculus I  
12 December 2019, Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

---

**Q-1)** Find the volume of the solid obtained by rotating around the  $y$ -axis the region bounded by the lines  $y = 4x - 12$ ,  $y = 15 - 5x$ ,  $x = 1$  and  $x = 6$ .

**Solution:**

You must notice that when  $1 \leq x \leq 3$ , the line  $y = 15 - 5x$  is above the other line, and on  $3 \leq x \leq 6$ , the line which is above the other one is  $y = 4x - 12$ . Hence the volume is calculated as follows.

$$\begin{aligned} V &= 2\pi \int_1^3 x[(15 - 5x) - (4x - 12)] dx + 2\pi \int_3^6 x[(4x - 12) - (15 - 5x)] dx \\ &= 2\pi \int_1^3 (27x - 9x^2) dx + 2\pi \int_3^6 (9x^2 - 27x) dx \\ &= 2\pi \left( \frac{27}{2}x^2 - 3x^3 \Big|_1^3 \right) + 2\pi \left( 3x^3 - \frac{27}{2}x^2 \Big|_3^6 \right) \\ &= 2\pi(30) + 2\pi\left(\frac{405}{2}\right) \\ &= 60\pi + 405\pi \\ &= 465\pi. \end{aligned}$$