Bilkent University

Quiz # 03
Math 101-Section 12 Calculus I
18 October 2020 Sunday
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Solution Key

Q-1) The height, width and the length of a rectangular solid, given as functions of time are $x(t)$, $y(t)$ and $z(t)$. At a certain time $t = t_0$ we observe that

$$x(t_0) = 4\,cm, \qquad y(t_0) = 3\,cm, \qquad z(t_0) = 2\,cm,$$
$$x'(t_0) = 1\,cm/sec, \qquad y'(t_0) = -2\,cm/sec, \qquad z'(t_0) = 1\,cm/sec.$$

(a) How fast is the volume of this solid changing at $t = t_0$?
(b) How fast is the surface area of this solid changing at $t = t_0$?

Solution:

(a) Volume is $V(t) = x(t)y(t)z(t)$. Then we have at $t = t$

$$\begin{aligned} V'(t) &= x'(t_0)y(t_0)z(t_0) + x(t_0)y'(t_0)z(t_0) + x(t_0)y(t_0)z'(t_0) \\ &= (1)(3)(2) + (4)(-2)(2) + (4)(3)(1) \\ &= 2. \end{aligned}$$

At that time the volume is **increasing** at a rate of $2m^2/sec$.

(b) The surface area of the solid is $S(t) = 2(x(t)y(t) + x(t)z(t) + y(t)z(t))$. Then we have at $t = t_0$

$$\begin{aligned} S'(t_0) &= 2(x'(t_0)y(t_0) + x(t_0)y'(t_0) + x'(t_0)z(t_0) + x(t_0)z'(t_0) + y'(t_0)z(t_0) + y(t_0)z'(t_0)) \\ &= 2((1)(3) + (4)(-2) + (1)(2) + (4)(1) + (-2)(2) + (3)(1)) \\ &= 0. \end{aligned}$$

At that time the surface area of the solid is **not changing**.