Bilkent University

Quiz # 04
Math 101-Section 12 Calculus I
1 November 2020 Sunday
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Solution Key

Q-1) Consider the polynomial $f(x) = 4x^5 - 15x^4 + 20x^3 - 30x^2 + 40x - 10$.

(i) Use the Intermediate Value Theorem (IVT) to show that $f(x) = 0$ has at least three solutions.

(ii) Use Rolle's theorem to show that $f(x) = 0$ has exactly three solutions.

Hint: $f'(x)$ can be easily factored.

Solution: **(i)** We try some values for x:

$$f(0) = -10, f(1) = 9, f(2) = -2, f(3) = 137.$$

There are three sign changes. f is continuous. Therefore there are at least three real roots of f on the interval $(0, 3)$.

(ii) By Rolle's theorem, between any two roots of f, there is a root of f'. If f has more than three roots, then f' will have more than two roots. But

$$f'(x) = 20x^4 - 60x^3 + 60x^2 - 60x + 40,$$

and by trial and error we find that

$$f(1) = 0, f(2) = 0.$$

This means $(x-1)(x-2)$ divides f. Hence we find that

$$f'(x) = 20(x-1)(x-2)(x^2+1),$$

which has only two real roots. Hence f cannot have more than three roots.

Here is the graph of $y = f(x)$ for your information. This was not required in this quiz.