Bilkent University

Quiz # 06
Math 101-Section 12 Calculus I
18 November 2021 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1)

(a) Write an equation for the tangent line to the curve $y = 1 - x^2$ at $x = a$, where $a > 0$.

(b) Find the area of the triangle formed by this tangent line, the x-axis and the y-axis.

(c) Find the minimum value of this area.

Show your work. Simplify as much as possible.

Grading: 2+2+6 points

Solutions:

(a) $y' = -2x$ so at $x = a$ the slope of the tangent line is $-2a$. An equation for the tangent line is then $y - (1 - a^2) = (-2a)(x - a)$, or after simplification

$$y = -2ax + a^2 + 1.$$

(b) This tangent line intersects the x-axis at the point $(\frac{a^2+1}{2a}, 0)$ and the y-axis at $(0, a^2 + 1)$. Then the area of the mentioned triangle is

$$A(a) = \frac{1}{2}\frac{a^2+1}{2a}(a^2+1) = \frac{(a^2+1)^2}{4a}, \ a > 0.$$

(c) We calculate to find

$$A'(a) = \frac{(3a^2-1)(a^2+1)}{4a^2}.$$

Then $A'(a) = 0$ when $a = 1/\sqrt{3}$. (Note $a > 0$.)

Now either by checking the sign change of $A'(a)$ at $a = 1/\sqrt{3}$ or noticing that

$$A''(a) = \frac{3a^4+1}{2a^3} > 0 \ \text{ when } a > 0,$$

we conclude that $a = 1/\sqrt{3}$ gives the minimum value of $A(a)$.

Finally the minimum value of the area is

$$A\left(\frac{1}{\sqrt{3}}\right) = \frac{4\sqrt{3}}{9}.$$