Bilkent University

Quiz # 08
Math 101-Section 12 Calculus I
2 December 2021 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

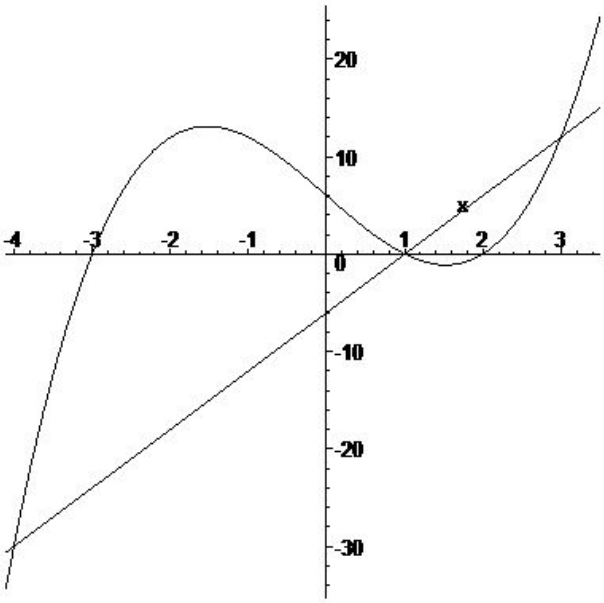
Q-1) Find the area bounded by the curves $y = x^3 - 7x + 6$ and $y = 6x - 6$.

Show your work. Simplify as much as possible.

Hint: $x^3 - 7x + 6 = (x-1)(x-2)(x+3)$.

Solutions:

Here is a sketch of the curves:



Solving $(x-1)(x-2)(x+3) = 6(x-1)$ gives $x = 1$, $x = 3$ and $x = -4$ as the x-coordinates of the intersection points of these curves. Then the area can be easily calculated as follows.

$$\begin{aligned}
A &= \int_{-4}^{1} [(x^3 - 7x + 6) - (6x - 6)]\, dx + \int_{1}^{3} [(6x - 6) - (x^3 - 7x + 6)]\, dx \\
&= \int_{-4}^{1} [x^3 - 13x + 12]\, dx - \int_{1}^{3} [x^3 - 13x + 12]\, dx \\
&= \left(\frac{1}{4}x^4 - \frac{13}{2}x^2 + 12x \Big|_{-4}^{1} \right) - \left(\frac{1}{4}x^4 - \frac{13}{2}x^2 + 12x \Big|_{1}^{3} \right) \\
&= \frac{375}{4} - (-8) \\
&= \frac{407}{4}.
\end{aligned}$$