Bilkent University

Quiz # 01
Math 101-Section 08 Calculus I
07 October 2022 Friday
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Solution Key

Q-1) Let f be defined as

$$f(x) = \begin{cases} 4x^2 + x + 7, & x < 3 \\ Ax + B, & x \geq 3. \end{cases}$$

Assuming that f is differentiable everywhere, find A and B.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 10 points.

Solution: Since f is differentiable then it must be continuous in particular at $x = 3$. This means that the limit of f as x approaches to 3 exists and is $f(3)$. Since this limit exists, the left limit also exists and is equal to the limit itself. This gives

$$f(3) = \lim_{x \to 3} f(x) = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (4x^2 + x + 7) = 46.$$

Since we have $f(3) = 3A + B$, the first identity we get is

$$3A + B = 46.$$

Since f is differentiable at $x = 3$, the right and left derivatives should exist and be equal. We then have

$$f'_-(3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3^-} \frac{4x^2 + x + 7 - 46}{x - 3} = \lim_{x \to 3^-} \frac{(x-3)(4x+13)}{x - 3} = 25.$$

Also

$$f'_+(3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3^+} \frac{Ax + B - (3A + B)}{x - 3} = \lim_{x \to 3^+} \frac{A(x-3)}{x - 3} = A.$$

Thus our second equation is

$$A = 25.$$

Putting this into our first equation we find

$$B = -29.$$