Bilkent University

Quiz # 02
Math 101-Section 12 Calculus I
13 October 2022 Thursday
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Solution Key

Q-1) Let f and g be defined as

$$f(x) = x^4 + x^3 - x^2 - x + 5, \quad g(x) = x^5 + x + 1.$$

Calculate the following values using chain rule and clearly indicating each step.

(a) $(f \circ g)'(1)$

(b) $(g \circ f)'(0)$.

(c) $(f \circ f)'(1)$.

(d) $(g \circ g)'(0)$.

(e) Assuming that h is differentiable everywhere write $(h \circ h \circ h)'(x)$ in terms of h and h'.

(f) Assuming that h and k are differentiable everywhere, write $(h \circ k)''(x)$ in terms of h, k and their derivatives.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 1+1+1+1+3+3=10 points.

Solution: We first calculate some values of f, g, f' and g'.

$f(0) = 5$	$f(1) = 5$	$g(1) = 3$	$g(-1) = -1$	$g(0) = 1$
$f'(3) = 128$	$f'(0) = -1$	$f'(1) = 4$	$f'(5) = 564$	$f'(-1) = 0$
$g'(1) = 6$	$g'(-1) = 6$	$g'(5) = 3126$	$g'(0) = 1$	$g'(3) = 406$

(a) $(f \circ g)'(1) = f'(g(1))g'(1) = f'(3)g'(1) = 128 \cdot 6 = 768.$

(b) $(g \circ f)'(0) = g'(f(0))f'(0) = g'(5)f'(0) = 3126 \cdot (-1) = -3126.$

(c) $(f \circ f)'(1) = f'(f(1))f'(1) = f'(5)f'(1) = 564 \cdot 4 = 2256.$

(d) $(g \circ g)'(0) = g'(g(0))g'(0) = g'(1)g'(0) = 6 \cdot 1 = 6.$

(e) $(h \circ h \circ h)'(x) = h'(h(h(x)))\, h'(h(x))\, h'(x).$

(f)

$$\begin{aligned}
(h \circ k)'(x) &= h'(k(x))\, k'(x) = (h' \circ k)(x)\, k'(x) \\
(h \circ k)''(x) &= h''(k(x))\, k'(x)\, k'(x) + (h' \circ k)(x)\, k''(x) \\
&= h''(k(x))(k'(x))^2 + h'(k(x))k''(x).
\end{aligned}$$