Bilkent University

Quiz # 04
Math 101-Section 12 Calculus I
3 November 2022 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the function

$$f(x) = \frac{x}{x^2 - 9}.$$

(a) Find the asymptotes of the graph $y = f(x)$, if any.

(b) Find f' and the critical points.

(c) Find f''.

(d) Prepare a table of the relevant values of f, f', f'' and show intervals of increase/decrease, concave up/down, etc

(e) Sketch the graph of $y = f(x)$.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 2+2+1+3+2=10 points.

Solution:

(a) There is a vertical asymptote at $x = -3$ and at $x = 3$.

(b) $f'(x) = -\dfrac{x^2 + 9}{(x^2 - 9)^2}$. There are no critical points. (It is alright if you call $x = \pm 3$ as critical points.)

(c) $f''(x) = \dfrac{2x(x^2 + 27)}{(x^2 - 9)^3}$.

(d)

	∞	-3	0	3	∞
f	$-$	$+$	$-$	$+$	
f'	$-$	$-$	$-$	$-$	
f''	$-$	$+$	$-$	$+$	
	$\searrow$	$\searrow$	$\searrow$	$\searrow$	
	$\frown$	$\smile$	$\frown$	$\smile$	

(e)