Bilkent University

Quiz # 05
Math 101-Section 12 Calculus I
10 November 2022 Thursday
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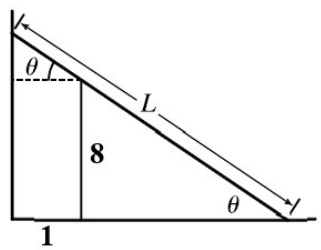
Solution Key

Q-1) A fence 8 m tall runs parallel to a tall building at a distance of 1 m from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?
Show your work in detail. Correct answers without detailed explanation do not get any credit.



Grading: 10 points.

Solution:

From the figure we have
$$L = \frac{8}{\sin\theta} + \frac{1}{\cos\theta}, \quad 0 < \theta < \frac{\pi}{2}.$$
We then find
$$L'(\theta) = -\frac{8\cos\theta}{\sin^2\theta} + \frac{\sin\theta}{\cos^2\theta}.$$
We see that $L'(\theta) = 0$ is equivalent to $\tan^3\theta = 8$, or $\tan\theta = 2$.

For this particular θ we have $\sin\theta = \dfrac{2}{\sqrt{5}}$ and $\cos\theta = \dfrac{1}{\sqrt{5}}$.

Then we find $L(\theta) = 5\sqrt{5} \approx 11$ m 18 cm for this critical value of θ.

We check that
$$\lim_{\theta\to 0^+} L(\theta) = \lim_{\theta\to(\pi/2)^-} L(\theta) = \infty.$$
Hence the above value of $L(\theta)$ gives the shortest ladder required in the problem.

You can also check the sign change of $L'(\theta)$ at the critical point and use the first derivative test to conclude that $\theta = \arctan 2$ gives the minimum value for $L(\theta)$.

Or if you are patient you can calculate $L''(\theta)$ and observe that it is positive hence the above critical value is minimum by the second derivative test.