Bilkent University

Quiz # 06
Math 101-Section 08 Calculus I
18 November 2022 Friday
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Solution Key

Q-1) Evaluate only the integral in (iii).

(i) Express the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+2i}$ as a definite integral on the interval $[0,1]$.

(ii) Express the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+2i}$ as a definite integral on the interval $[0,2]$.

(iii) Express the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\frac{i}{n}}{n}$ as a definite integral on the interval $[0,1]$.

(iv) Express the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\frac{i}{n}}{n}$ as a definite integral on the interval $[0,7]$.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 2+2+(2+2)+2=10 points.

Solution:

(i) $$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+2i}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\frac{1}{1+2\frac{i}{n}}=\int_0^1\frac{dx}{1+2x}.$$

(ii) $$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n+2i}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{2}\frac{2}{n}\frac{1}{1+\frac{2i}{n}}=\frac{1}{2}\int_0^2\frac{dx}{1+x}.$$

(iii) $$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\frac{i}{n}}{n}=\int_0^1\cos x\,dx=\left(\sin x\Big|_0^1\right)=\sin 1.$$

(iv) $$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\cos\frac{i}{n}}{n}=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{7}\frac{7}{n}\cos(\frac{1}{7}\frac{7i}{n})=\frac{1}{7}\int_0^7\cos\frac{x}{7}\,dx.$$