Bilkent University

Quiz # 08
Math 101-Section 08 Calculus I
2 December 2022 Friday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ around the x-axis using two different methods: (i) disc method and (ii) cylindrical shells method. Here $a, b > 0$.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+5=10 points.

Hint: Make sure that you get the same volume in both parts. Also make sure that you get the volume of a sphere when $a = b$. ☺

Solution:

(i) The disc method: If we set $y = f(x)$ for this ellipse in the upper half plane, i.e. for $y \geq 0$, then we have

$$f(x) = \frac{b}{a}\sqrt{a^2 - x^2}, \quad \text{where} \quad x \in [-a, a].$$

Then the volume is given, from symmetry, as:

$$\begin{aligned} V &= 2\left[\pi \int_0^a f(x)^2 dx\right] \\ &= \frac{2\pi b^2}{a^2}\int_0^a (a^2 - x^2)dx \\ &= \frac{2\pi b^2}{a^2}\left(a^2 x - \frac{x^3}{3}\Big|_0^a\right) \\ &= \frac{4\pi}{3}ab^2. \end{aligned}$$

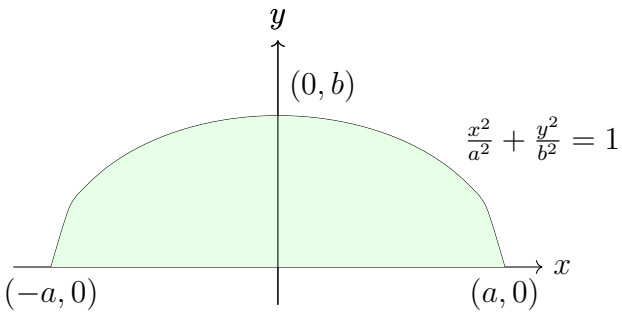
(ii) The cylindrical shells method: If we set $x = f(y)$ for this ellipse in the right half plane, i.e. for $x \geq 0$, then we have

$$f(y) = \frac{a}{b}\sqrt{b^2 - y^2}, \quad \text{where} \quad y \in [-b, b].$$

Then the volume is given, from symmetry, as:

$$\begin{aligned} V &= 2\left[2\pi \int_0^b y f(y) dy\right] \\ &= \frac{4\pi a}{b}\int_0^b y\sqrt{b^2 - y^2}dy \\ &= \frac{4\pi a}{b}\left(-\frac{(b^2 - y^2)^{3/2}}{3}\Big|_0^b\right) \\ &= \frac{4\pi}{3}ab^2. \end{aligned}$$

Check that when $a = b = r$ we get the volume of a sphere with radius r.



In this problem we are revolving the shaded region around the x-axis.