



Bilkent University

Quiz # 10
Math 101-Section 08 Calculus I
16 December 2022 Friday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Calculate the following:

(i) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x - x \cos x}$.

(ii) $\int x^2 \ln x \, dx$.

(iii) $\int_1^{\infty} \frac{\ln x}{x^2} \, dx$.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 3+3+4=10 points.

Solution:

(i)

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x - x \cos x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - \cos x + x \sin x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\sin x}{2 \sin x + x \cos x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\cos x}{3 \cos x - x \sin x} = \frac{1}{3}.$$

(ii) Use integration by-parts with $u = \ln x$ and $dv = x^2$ to get

$$\int x^2 \ln x \, dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 \, dx = \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C.$$

(iii) First use integration by-parts with $u = \ln x$ and $dv = \frac{dx}{x^2}$ to get

$$\int \frac{\ln x}{x^2} \, dx = -\frac{\ln x}{x} + \int \frac{dx}{x^2} = -\frac{\ln x}{x} - \frac{1}{x} + C.$$

Since we have $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$ and $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$, we get

$$\int_1^{\infty} \frac{\ln x}{x^2} \, dx = \left(-\frac{\ln x}{x} - \frac{1}{x} \Big|_1^{\infty} \right) = 1.$$