Bilkent University

Quiz # 01
Math 101-Section 04 Calculus I
28 September 2023 Thursday
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Solution Key

Q-1) Find all values of a and b which make the following function continuous everywhere.

$$f(x)=\begin{cases} ax^3 & x<2 \\ ax^2+b & 2\le x<4 \\ x+3b & 4\le x \end{cases}$$

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+5=10 points if satisfactory explanations are provided.

Solution:

At all points except at $x=2$ and $x=4$ our function is given by a polynomial and hence is continuous. We need to check continuity only at $x=2$ and $x=4$.

At $x=2$ we must have

$$\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^+} f(x) \\ \lim_{x\to 2^-} ax^3 &= \lim_{x\to 2^+} (ax^2+b) \\ 8a &= 4a+b, \end{aligned}$$

which gives $b=4a$. Moreover at $x=4$ we must have

$$\begin{aligned} \lim_{x\to 4^-} f(x) &= \lim_{x\to 4^+} f(x) \\ \lim_{x\to 4^-} (ax^2+b) &= \lim_{x\to 4^+} (x+3b) \\ 16a+b &= 4+3b, \end{aligned}$$

which gives $16a=4+2b$. Using the previous equality $b=4a$ we find $a=1/2$ and $b=2$.

Hence the answer is $a=1/2$ and $b=2$ are the only values of a and b making f continuous everywhere. Here is a graph of this function: