Quiz # 01
Math 101-Section 05 Calculus I
28 September 2023 Thursday
Instructor: Ali Sinan Sertöz
**Solution Key**

Bilkent University

**Q-1)** Find all values of $a$ and $b$ which make the following function continuous everywhere.

$$f(x) = \begin{cases} ax^2 & x < 2 \\ ax + b & 2 \leq x < 5 \\ x^2 + b & 5 \leq x \end{cases}$$

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+5=10 points if satisfactory explanations are provided.

**Solution:**

At all points except at $x = 2$ and $x = 5$ our function is given by a polynomial and hence is continuous. We need to check continuity only at $x = 2$ and $x = 5$.

At $x = 2$ we must have

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$
$$\lim_{x \to 2^-} ax^2 = \lim_{x \to 2^+} (ax + b)$$
$$4a = 2a + b,$$

which gives $b = 2a$. Moreover at $x = 5$ we must have

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x)$$
$$\lim_{x \to 5^-} (ax + b) = \lim_{x \to 5^+} (x^2 + b)$$
$$5a + b = 25 + b,$$

which gives $a = 5$. Using the previous equality $b = 2a$ we find $b = 10$.

Hence the answer is $a = 5$ and $b = 10$ are the only values of $a$ and $b$ making $f$ continuous everywhere. Here is a graph of this function: