Bilkent University

Quiz # 04
Math 101-Section 05 Calculus I
19 October 2023 Thursday
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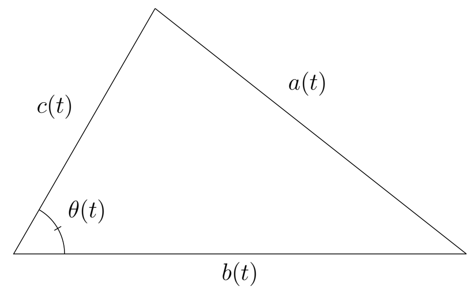
Solution Key

Q-1)

The sides of the triangle on the right are changing as differentiable functions of time. At a particular time, say at $t = t_0$, we observe that $b(t_0) = 8$cm, $c(t_0) = 5$cm and $\theta(t_0) = \pi/3$. We also observe that at that moment side a is increasing at a rate of 2cm/s, side b is increasing at a rate of 1cm/s and side c is decreasing at a rate of 1cm/s. Find how fast θ is changing at that moment.



Hint: You may find it useful to recall the cosine rule $a^2 = b^2 + c^2 - 2bc\cos\theta$.

Grading: 10 points

Solution:

From the cosine rule we find that

$$a(t)^2 = b(t)^2 + c(t)^2 - 2b(t)c(t)\cos\theta(t), . \qquad (*)$$

Substituting in the values $b(t_0) = 8$cm, $c(t_0) = 5$cm and $\theta(t_0) = \pi/3$ we find that

$$a(t_0) = 7cm$$

Taking derivatives of both sides of $(*)$ with respect to t we find

$$\begin{aligned} 2a(t_0)a'(t_0) =& 2b(t_0)b'(t_0) + 2c(t_0)c'(t_0) - 2b'(t_0)c(t_0)\cos\theta(t_0) - b(t_0)c'(t_0)\cos\theta(t_0) \\ &+ 2b(t_0)c(t_0)\sin\theta(t_0)\theta'(t_0). \end{aligned}$$

Again putting in the given data $a(t_0) = 7$cm, $b(t_0) = 8$cm, $c(t_0) = 5$cm, $\theta(t_0) = \pi/3$, $a'(t_0) = 2$cm/s, $b'(t_0) = 1$cm/s, $c'(t_0) = -1$cm/s and recalling that $\cos\pi/3 = 1/2$ and $\sin\pi/3 = \sqrt{3}/2$, we find that

$$\theta'(t_0) = \frac{19\sqrt{3}}{120}cm/s.$$

Hence θ is **increasing** at the rate of $\dfrac{19\sqrt{3}}{120}$cm/s at that moment. (This is approximately 3mm/s.)