Bilkent University

Quiz # 06
Math 101-Section 04 Calculus I
2 November 2023 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the function $f(x) = (2x + 7)(x - 1)^2$ on the interval $[-4, 2]$.

(a) Find the critical points of f and decide if f has a local minimum or local maximum values at these critical points.

(b) Find the global minimum and the global maximum values of f.

(c) Find the inflection points of f.

(d) Sketch the graph of $y = f(x)$. No explanation is necessary. Just make sure that the concavity, intercepts etc are clearly visible in your sketch.

Grading: 4+2+1+3=10 points

Solution: (Grader: `taha.yigit@ug.bilkent.edu.tr`)

(a) $f(x) = 2x^3 + 3x^2 - 12x + 7$ and $f'(x) = 6x^2 + 6x - 12 = 6(x - 1)(x + 2)$. Hence the critical points, from $f'(x) = 0$, are $x = 1$ and $x = -2$.

$f'(x)$ changes sign from + to – at $x = -2$, so this is a local maximum point. Or alternatively you may consider $f''(x) = 6(2x + 1)$ which is negative at $x = -2$. This says that $x = -2$ is a local maximum point for f.

Similarly $f'(x)$ changes sign from – to + at $x = 1$, so this is a local minimum point. Or alternatively you may consider $f''(x) = 6(2x + 1)$ which is positive at $x = 1$. This says that $x = 1$ is a local minimum point for f.

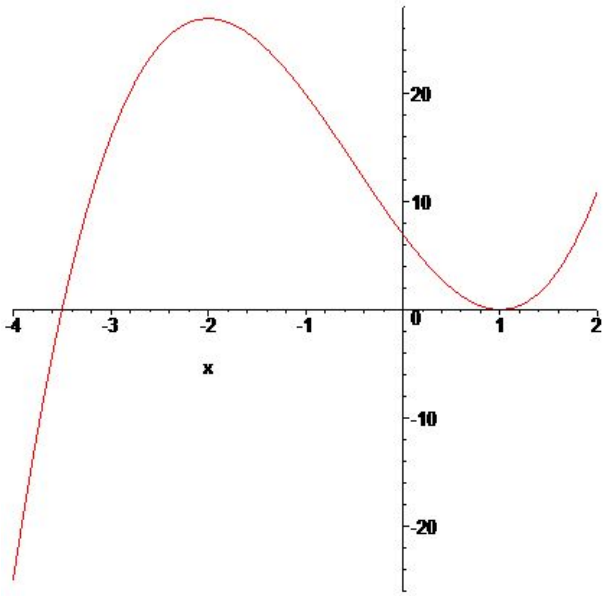
(b) We evaluate f at the critical and end points.

$$f(-4) = -25,\ f(-2) = 27,\ f(1) = 0,\ f(2) = 11.$$

Hence the global maximum value of f is 27 achieved at $x = -2$, and the global minimum value is -25 achieved at $x = -4$.

(c) $f''(x) = 6(2x + 1) = 0$ gives $x = -1/2$. We check that $f''(x)$ changes sign at this point so it is the only inflection point.

(d) Here is a graph of $y = f(x)$.


20
10
-4
-3
-2
-1
0
1
2
x
-10
-20