



Bilkent University

Quiz # 06
Math 101-Section 05 Calculus I
2 November 2023 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the function $f(x) = (2x + 7)(x - 1)^2$ on the interval $[-4, 2]$.

- (a) Find the critical points of f and decide if f has a local minimum or local maximum values at these critical points.
- (b) Find the global minimum and the global maximum values of f .
- (c) Find the inflection points of f .
- (d) Sketch the graph of $y = f(x)$. No explanation is necessary. Just make sure that the concavity, intercepts etc are clearly visible in your sketch.

Grading: $4+2+1+3=10$ points

Solution: (Grader: rbulakguler71@gmail.com)

(a) $f(x) = 2x^3 + 3x^2 - 12x + 7$ and $f'(x) = 6x^2 + 6x - 12 = 6(x - 1)(x + 2)$. Hence the critical points, from $f'(x) = 0$, are $x = 1$ and $x = -2$.

$f'(x)$ changes sign from $+$ to $-$ at $x = -2$, so this is a local maximum point. Or alternatively you may consider $f''(x) = 6(2x + 1)$ which is negative at $x = -2$. This says that $x = -2$ is a local maximum point for f .

Similarly $f'(x)$ changes sign from $-$ to $+$ at $x = 1$, so this is a local minimum point. Or alternatively you may consider $f''(x) = 6(2x + 1)$ which is positive at $x = 1$. This says that $x = 1$ is a local minimum point for f .

(b) We evaluate f at the critical and end points.

$$f(-4) = -25, f(-2) = 27, f(1) = 0, f(2) = 11.$$

Hence the global maximum value of f is 27 achieved at $x = -2$, and the global minimum value is -25 achieved at $x = -4$.

(c) $f''(x) = 6(2x + 1) = 0$ gives $x = -1/2$. We check that $f''(x)$ changes sign at this point so it is the only inflection point.

(d) Here is a graph of $y = f(x)$.

