Bilkent University

Quiz # 10
Math 101-Section 04 Calculus I
30 November 2023 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1)

(a) Calculate $\lim_{x \to 0} \frac{\ln(x+1)}{x}$ without using L'Hospital's rule.

(b) Evaluate $\int_0^{\pi/2} e^{\sin x} \sin 2x \, dx$.

Hint: What is $\frac{d}{dt}[e^t(t-1)] = ?$

(c) Show that the tangent line to the curve $y = e^x$ at $x = 2024$ intersects x-axis at $x = 2023$.

Grading: 3+4+3=10 points

Solution: (Grader: taha.yigit@ug.bilkent.edu.tr)

(a)

$$\lim_{x \to 0} \frac{\ln(x+1)}{x} = \lim_{x \to 0} \frac{\ln(x+1) - \ln 1}{x} = \frac{d}{dt}\bigg|_{t=1} \ln t = \frac{1}{t}\bigg|_{t=1} = 1.$$

(b) Using the hint we know that

$$\frac{d}{dt}[e^t(t-1)] = e^t, \quad \text{i.e. in particular} \quad \int e^x x \, dx = xe^x - e^x + C.$$

Now we can evaluate the given integral.

$$\begin{aligned} \int_0^{\pi/2} e^{\sin x} \sin 2x \, dx &= 2 \int_0^{\pi/2} e^{\sin x} \sin x \cos x \, dx \\ &= 2 \int_0^1 e^u u \, du, \quad \text{where we put } u = \sin x \\ &= 2 \left(ue^u - e^u \Big|_{u=0}^{u=1} \right) \\ &= 2. \end{aligned}$$

(c) Since $(e^x)' = e^x$, an equation for the tangent line to $y = e^x$ at $x = 2024$ is of the form

$$L(x) = e^{2024}(x - 2024) + e^{2024} = e^{2024}(x - 2023),$$

and this line intersects the x-axis at $x = 2023$.