Quiz # 01
Math 101 Section 03 Calculus I
2 October 2024 Wednesday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Q-1) For a and b being some real numbers we define

$$f(x) = \begin{cases} \dfrac{x^2+ax+b}{x^2+x-6} & x \neq -3 \\ \\ \dfrac{8}{5} & x = -3. \end{cases}$$

(i) Find a relation between a and b so that $\lim_{x\to -3} f(x)$ exists.

(ii) Using (i) above write b in terms of a and after substituting this into $f(x)$ find a such that $f(x)$ becomes continuous at $x = -3$.

(iii) Now also find b.

Grading: 4+5+1=10 points

Solution: Grader: `gunes.akbas@bilkent.edu.tr`

(i) The denominator $x^2+x-6 = (x+3)(x-2)$ vanishes at $x=-3$. For the above limit to exist the numerator should also vanish at $x=-3$. This gives

$$x^2+ax+b\Big|_{x=-3} = 9-3a+b = 0, \quad \text{which is the required relation between } a \text{ and } b.$$

(ii) From (i) above we have $b = 3a-9$. Substituting this into the numerator we get

$$x^2+ax+3a-9 = (x^2-9)+(ax+3a) = (x-3)(x+3)+a(x+3) = (x+3)(x-3+a).$$

Then when $x \neq -3$ we have

$$f(x) = \frac{(x+3)(x-3+a)}{(x+3)(x-2)} = \frac{x-3+a}{x-2}.$$

For $f(x)$ to be continuous at $x=-3$ we must have $\lim_{x\to -3} f(x) = f(-3)$. Now $\lim_{x\to -3} f(x) = \dfrac{8}{5}$ gives $\dfrac{6-a}{5} = \dfrac{8}{5}$, which in turn gives $a=-2$.

(iii) From (i) above we have $b = 3a-9$. Putting in $a=-2$ we get $b=-15$.