Quiz # 01
Math 101 Section 07 Calculus I
3 October 2024 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Q-1) For a and b being some real numbers we define

$$f(x) = \begin{cases} \dfrac{x^2+ax+b}{x^2+x-6} & x \neq -3 \\ \\ 2 & x = -3. \end{cases}$$

(i) Find a relation between a and b so that $\lim_{x\to -3} f(x)$ exists.

(ii) Using (i) above write b in terms of a and after substituting this into $f(x)$ find a such that $f(x)$ becomes continuous at $x = -3$.

(iii) Now also find b.

Grading: 4+5+1=10 points

Solution: Grader: `gunes.akbas@bilkent.edu.tr`

(i) The denominator $x^2 + x - 6 = (x+3)(x-2)$ vanishes at $x = -3$. For the above limit to exist the numerator should also vanish at $x = -3$. This gives

$$x^2 + ax + b\Big|_{x=-3} = 9 - 3a + b = 0, \quad \text{which is the required relation between } a \text{ and } b.$$

(ii) From (i) above we have $b = 3a - 9$. Substituting this into the numerator we get

$$x^2 + ax + 3a - 9 = (x^2 - 9) + (ax + 3a) = (x-3)(x+3) + a(x+3) = (x+3)(x-3+a).$$

Then when $x \neq -3$ we have

$$f(x) = \frac{(x+3)(x-3+a)}{(x+3)(x-2)} = \frac{x-3+a}{x-2}.$$

For $f(x)$ to be continuous at $x = -3$ we must have $\lim_{x\to -3} f(x) = f(-3)$. Now $\lim_{x\to -3} f(x) = 2$ gives $\dfrac{6-a}{5} = 2$, which in turn gives $a = -4$.

(iii) From (i) above we have $b = 3a - 9$. Putting in $a = -4$ we get $b = -21$.